NEW PERSPECTIVES

8th to the 12th of July 2024

CIEM
Castro
Urdiales

IN BANACH SPACES AND BANACH LATTICES

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MINICOURSES

Antonio Avilés
(University of Murcia, Spain)

Valentin Ferenczi
(University of São Paulo, Brazil)

Eva Kopecká
(University of Innsbruck, Austria)

Piotr Koszmider
(Polish Academy of Sciences, Poland)

MAIN SPEAKERS

Luciano Abadías
(University of Zaragoza, Spain)

Ramón J. Aliaga
(Polytechnic University of Valencia, Spain)

Jesús M. F. Castillo
(University of Extremadura, Spain)

Michal Doucha
(Czech Academy of Sciences, Czech Republic)

Audrey Fovelle
(University of Granada, Spain)

Antonio J. Guirao
(Polytechnic University of Valencia, Spain)

Mingu Jung
(KIAS, South Korea)

José Orihuela
(University of Murcia, Spain)

Alicia Quero
(ČVUT, Czech Republic)

Tommaso Russo
(University of Innsbruck, Austria)

Richard J. Smith
(University College Dublin, Ireland)

ORGANIZING COMMITTEE

Luis C. García-Lirola (UZ)
Sheldon Gil Dantas (UV)
Gonzalo Martínez Cervantes (UM)

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Excmo. Ayuntamiento de Castro Urdiales
Instituto Universitario de Investigación de Matemáticas y Aplicaciones Universidad Zaragoza
Departamento de Matemáticas Universidad Zaragoza
Universidad de Murcia | Departamento de Matemáticas

About the Summer School 2024

NEW PERSPECTIVES IN BANACH SPACES AND BANACH LATTICES

The School is aimed at researchers and PhD students in Banach Space Theory and related areas. The comprehensive program encompasses a series of mini-courses and talks by both junior and senior researchers, as well as free time to foster open discussions and collaboration among participants.

Location

The School will be held at the Centro Internacional de Encuentros Matemáticos (CIEM) in Castro Urdiales (Spain).

School dinner will be held at **Sidrería Marcelo** on Wednesday evening.

Organizing committee

- Luis C. García Lirola (Universidad de Zaragoza)
- Sheldon Gil Dantas (Universitat de València)
- Gonzalo Martínez Cervantes (Universidad de Murcia)

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00 - 10:00	Registration and opening E. Kopecká	A. Avilés	V. Ferenczi	E. Kopecká	A. Avilés
10:00 - 11:00	A. Fovelle	P. Koszmider Break	A. Avilés Break	P. Koszmider Break	V. Ferenczi Break
11:00 - 12:00	Break V. Ferenczi	E. Kopecká	P. Koszmider	A. Guirao	A. Quero
12:00 - 13:00	R. Aliaga	E. Martínez T. Veeorg	M. Jung B. Farkas	M. Doucha R. Medina	J. Orihuela J. Castillo
13:00 - 14:00	R. Smith Lunch	V. Olmos Lunch	S. Basu Lunch	M. Han Lunch	Lunch
14:00 - 15:00					
15:00 - 16:00	T. Russo	A. Quilis L. Sáenz		Godoy/Manzano Manev/Kaasik	
16:00 - 17:00	D. Rodríguez J. Rondoš	D. Ravasini Problem session		Thimm/GarcíaB Cobollo	
17:00 - 18:00	D. Sobota			Lõo	

Abstracts

Minicourses

Banach lattices

Antonio Avilés
Universidad de Murcia

Tuesday 9:10 - 9:50 **Wednesday**: 10.00 - 10.40 **Friday**: 9.10 - 9.50

We will review some recent developments in the theory of Banach lattices.

Weak limits of consecutive projections onto convex sets

Eva Kopecká
Universität Innsbruck

Monday: 9.30 - 10.10 **Tuesday**: 11.15 - 11.55 **Thursday**: 9.10 - 9.50

We try to find a point in the intersection of closed convex sets by iterating the nearest point projection onto them.

Let $\{C_\alpha\}_{\alpha\in\Omega}$ be a family of closed and convex sets with non-empty intersection C in a Hilbert space H. A fixed sequence $\{\alpha(n)\} \subset \Omega$ and a starting element $x_0 \in H$ generate the sequence $x_{n+1} = P_{\alpha(n)}x_n$, $n = 0, 1, 2, \dots$, of consecutive projections that can be examined for convergence.

Even for finite Ω, there is in general no convergence in norm and the case of weak convergence is open. For finitely many closed, convex and *symmetric* sets there is always weak convergence. We will discuss this classical result.

Some additional properties of the sequence $\{\alpha(n)\} \subset \Omega$ of indices imply convergence. We will concentrate on the case of remotest projections, when in every step we project onto the most distant set C_α. For an arbitrarily large Ω, symmetric closed convex sets and remotest projection there is convergence in norm, and if we remove the symmetry assumption, there is weak convergence.

We also project onto almost remotest sets. This means, x_{n+1} is the projection of x_n onto such a set $C_{\alpha(n)}$ that the ratio of the distances from x_n to this set and to any other set from the family is at least $t_n \in [0, 1]$. We study properties of the weakness parameters t_n and of the sets C_α which ensure the norm or weak convergence of the sequence $\{x_n\}$ to a point in C.

Joint work with P. Borodin.

On Ramsey-type properties of the distance in nonseparable spheres

Piotr Koszmider

Institute of Mathematics of the Polish Academy of Sciences

Tuesday: 10.00 - 10.40   **Wednesday**: 11.15 - 11.45

Thursday: 10.00 - 10.40

Given an uncountable subset Y of the unit sphere of a nonseparable Banach space, is there an uncountable Z included in Y such that the distances between any two distinct points of Z are more or less the same? If an uncountable subset Y of such a sphere does not admit an uncountable Z included in Y, where any two points are distant by more than $r > 0$, is it because Y is the countable union of sets of diameters not bigger than r?

Clearly, these types of questions can be rephrased in the combinatorial language of partitions of pairs of points of a Banach space X induced by the distance function. We investigate connections between the set-theoretic phenomena involved (both descriptive and combinatorial) and the geometric properties of uncountable subsets of the unit spheres of nonseparable Banach spaces of densities up to continuum related to uncountable (1+)-separated sets, equilateral sets or Auerbach systems.

On topological and multidimensional aspects of Mazur rotations problem

Valentin Ferenczi

Universidade de São Paulo

Monday: 11.30 - 12.10   **Wednesday**: 9.10 - 9.50   **Friday**: 10.00 - 10.40

This minicourse will be concentrated on "large" actions of linear isometry groups on Banach spaces. We shall recall the isometry groups of some classical examples of Banach spaces, with Mazur rotations problem as our guideline, including the topological side of the question (topological amenability, extreme amenability). Special attention will be given to isometry groups on $C(K)$-spaces, with focus on the "oligomorphic" property of $C(2^\omega)$, and to isometry groups on the Lebesgue spaces $L_p(0,1)$, with focus on the multidimensional aspects of their action (ultrahomogeneity, approximate ultrahomogeneity, Fraïssé property).

Based on joint work in progress with Jordi Lopez-Abad (UNED Madrid).

Main speakers

Asymptotic smoothness and concentration properties

Monday
10.20-10.55

Audrey Fovelle
Universidad de Granada

In this talk, we will be interested in concentration properties for Lipschitz maps defined on Hamming graphs. After introducing all the objects at stake and explaining their interest, we will see how one can construct the first example of a Banach space that has some concentration without any smoothness property.

Lipschitz realcompactifications and functionals on Lipschitz spaces

Monday
12.20-12.55

Ramón J. Aliaga
Universitat Politècnica de València

The uniform (or Samuel) compactification $M^{\mathcal{U}}$ and the Lipschitz realcompactification $M^{\mathcal{R}}$ are extensions of a metric space (M, d) where Lipschitz functions can be extended continuously. We will review and discuss how they can be used as an ideal setting for the study of continuous linear functionals on Lipschitz spaces $\mathrm{Lip}_0(M)$, and for extension of known results about functionals in the Lipschitz-free space $\mathcal{F}(M)$. In particular, we will show how to properly define the support of such functionals and obtain a Lipschitz version of the Riesz-Markov-Kakutani representation theorem. We will also show how to extend the metric of M to the realcompactification, turning $M^{\mathcal{R}}$ into the "metric bidual" of M, and use this to characterize optimal measure-based representations of functionals on $\mathrm{Lip}_0(M)$ in terms of optimal transport in $M^{\mathcal{R}}$. This talk will be based on joint work with E. Pernecká and R. J. Smith.

Functionals on Lipschitz spaces and Choquet representation theory

Monday
13.00-13.35

Richard Smith
University College Dublin

Let $\mathrm{Lip}_0(M)$ denote the Banach space of real-valued Lipschitz functions on a complete metric space (M, d) that vanish at a point $0 \in M$. To date, there is no known general representation theorem for the dual space of functionals $\mathrm{Lip}_0(M)^*$ (which includes the Lipschitz-free space $\mathcal{F}(M)$ over M). The structure of $\mathrm{Lip}_0(M)^*$ can be probed using the 'De Leeuw transform', which yields (non-unique) representations of each functional in $\mathrm{Lip}_0(M)^*$ in the form of measures on the Stone-Čech compactification $\beta\widetilde{M}$ of $\widetilde{M} := \{(x, y) \in M \times M : x \neq y\}$. We use techniques from Choquet theory to find 'well-behaved' representations of a given functional, and apply our results to problems concerning the extremal structure of Lipschitz-free spaces.

This talk is based on joint work with Ramón Aliaga (Universitat Politècnica de València) and Eva Pernecká (Czech Technical University, Prague).

Can you tile the plane with closed balls?

Monday
15.30-16.05

Tommaso Russo
Universität Innsbruck

Is there a family $\mathcal{C}$ of closed balls in $\mathbb{R}^2$ with mutually disjoint interiors and whose union covers the plane? If we consider balls in the maximum norm on the plane the answer is quite clearly positive. But what happens for Euclidean balls? And is the answer based on 2-dimensional considerations?

A family $\mathcal{C}$ of closed convex sets with non-empty interior is a *tiling* of a normed space $\mathcal{X}$ if the convex sets in $\mathcal{C}$ have mutually disjoint interiors and cover $\mathcal{X}$. In the talk we will survey some classical and recent results concerning tilings of normed spaces. Among others, we shall explain a spectacular example, due to Klee, of a tiling of $\ell_1(\Gamma)$ with disjoint balls and we shall answer the question from the title. The talk is intended to be elementary and based on geometrical considerations (*a.k.a.* pictures).

Universality arising from composition operators

Luciano Abadías
Universidad de Zaragoza

A linear operator U acting boundedly on an infinite-dimensional separable complex Hilbert space H is universal if every linear bounded operator acting on H is similar to a scalar multiple of a restriction of U to one of its invariant subspaces. It turns out that characterizing the lattice of closed invariant subspaces of a universal operator is equivalent to solve the Invariant Subspace Problem for Hilbert spaces. In this talk, we consider some composition and weighted composition operators on spaces of measurable functions and on spaces of holomorphic functions, and we prove the universality of the translations by eigenvalues of such operators. Some consequences for the Banach space case are also discussed.

Strong subdifferentiable points in Lipschitz-free spaces

Wednesday
12.05-12.40

Mingu Jung
Korea Institute for Advanced Study

In this talk, we present some sufficient conditions on a metric space M for which every elementary molecule is a strongly subdifferentiable point in the Lipschitz-free space $\mathcal{F}(M)$. We will discuss one way of perturbing the metric space (M, d) to $(M, \tilde{d})$ in such a way that every finitely supported element in $\mathcal{F}(M, \tilde{d})$ is a strongly subdifferentiable point. This perturbation turns out to be a Lipschitz homeomorphism by nature when the given metric space (M, d) is uniformly discrete.

On projectional skeletons and the Plichko property in Lipschitz-Free Banach spaces

Thursday
11.15-11.50

Antonio J. Guirao
Universitat Politècnica de València

A Banach space X is said to be Plichko if there exists a pair (Δ, N) in $X \times X^*$ such that Δ is linearly dense in X, N is norming, and every element in N is countably supported on Δ. In the case where X is the Lipschitz-Free Banach space associated with a metric space M, a first simple question is: which geometrical property of M (if any) makes its $\mathcal{F}(M)$ a Plichko space where Δ can be taken as a subset of deltas (the isometric copy of M inside $\mathcal{F}(M)$)? A second, equally simple, question is whether such a geometrical characterization can be found for those metric spaces whose Lipschitz-Free space is Plichko but with every element in Δ being elementary molecules of $\mathcal{F}(M)$.

We present a joint work with V. Montesinos and A. Quilis, part of Quilis' PhD dissertation, which has been published in the Mediterranean Journal of Mathematics.

In particular, we study projectional skeletons and the Plichko property in Lipschitz-free spaces, relating these concepts to the geometry of the underlying metric space. We identify a metric property that characterizes the Plichko property witnessed by deltas in the associated Lipschitz-free space. Additionally, we show that the Lipschitz-free space of all $\mathbb{R}$-trees has the Plichko property witnessed by molecules. We define the concept of retractional trees to generalize this result to a larger class of metric spaces. Finally, we demonstrate that no separable subspace of ℓ_∞ containing c_0 is an r-Lipschitz retract for $r < 2$, which implies, in particular, that $\mathcal{F}(\ell_\infty)$ is not r-Plichko for $r < 2$.

Isometries of Lipschitz-free Banach spaces

Thursday
12.00-12.35

Michal Doucha
Czech Academy of Sciences

In my talk, I will give a complete description of surjective linear isometries and linear isometry groups of a fairly large class of Lipschitz-free Banach spaces, extending earlier results in this direction by Mayer-Wolf, Weaver, and Alexander, Fradelizi, García-Lirola, and Zvavitch. This includes Lipschitz-free spaces over any graph but also over some metric spaces containing large Euclidean subspaces such as non-abelian Carnot groups with horizontally strictly convex norms. I will also explore metric spaces, which we call Lipschitz-free rigid, whose Lipschitz-free space only admits surjective linear isometries coming from surjective dilations of the metric space itself. I will show that every metric space isometrically embeds into a Lipschitz-free rigid metric space with only three more points. This is joint work with Marek Cúth and Tamás Titkos.

Numerical index and the geometry of bounded linear operators

Friday
11.15-11.50

Alicia Quero
Czech Technical University in Prague

The numerical index of a given Banach space X is a constant that relates the usual norm to the numerical range of bounded linear operators on the space. This concept was introduced by Lumer in 1968 within the context of classifying operator algebras and has been widely studied since then as it describes the geometry of the space $\mathcal{L}(X)$ around the identity operator. With the purpose of characterizing in a similar way the geometry of the space of operators between two possibly different Banach spaces X and Y, Ardalani introduced the concept of numerical index with respect to an operator in 2014 by considering operators in $\mathcal{L}(X, Y)$ and fixing an arbitrary norm-one operator instead of the identity.

In this talk, we will provide an overview of the topic, analysing the differences and similarities between these concepts, and presenting both classical and recent results in this area.

Some of the results that will be presented are joint work with V. Kadets, M. Martín, J. Merí, and A. Pérez.

Variational Analysis and James's Theorem

Friday
12.00-12.35

José Orihuela
Universidad de Murcia

One of the most important achievements in optimisation is the James's weak compactness theorem. It says that a weakly closed subset A of a Banach space E is weakly compact if, and only if, every linear form $x^* \in E^*$ attains its supremum over A at some point of A. We present recent extensions of James's theorem. Among them we have: Let A be a closed, convex, bounded and not weakly compact subset of a Banach space E. Let us fix a convex and weakly compact subset W of E, a functional $z_0^* \in E^*$ and $\epsilon > 0$. Then there is a linear form $x_0^* \in B_{pW}(z_0^*, \epsilon)$, i.e.

$$|x_0^*(w) - z_0^*(w)| < \epsilon$$

for all $w \in W$, which does not attain its supremum on A. Moreover, if $z_0^*(A) < 0$ the same can be provided for the former non attaining linear form; i.e $x_0^*(A) < 0$ (one sided James's theorem). A multiset version of James's theorem is also true for weakly sequentially complete Banach spaces.

James's theorem is strongly connected with variational principles and optimisation theory. Indeed we will study unbounded versions of the former results leading us to the fact that for a weakly lower semicontinuous function $\alpha : E \longrightarrow (-\infty, +\infty]$, $\partial\alpha(E)$ has non empty interior for the Mackey topology if, and only if, the level sets $\{\alpha \leq c\}$ are weakly compact. It now follows that reflexive spaces are the only natural frame to develop variational analysis.

We shall continue our talk with $\sigma(E^*, E)$ versions of the formed results: Given

$$f : E^* \longrightarrow \mathbb{R} \cup \{+\infty\}$$

convex, proper and norm lower semicontinuous map on the dual E^* of a Banach space E such that:

$$\text{for every } x \in E, \quad x - f \text{ attains its supremum on } E^*;$$

i.e.

$$E \subset \partial f(E^*),$$

does it follow that f must be w^*-lower semicontinuous?

If E is assumed to be Asplund and weakly Lindelof determined the answer is yes. The same happens when we ask the lower semicontinuity of f for the topology of uniforme convergence on bounded and separable subsets of E, instead of the norm topology. Even more, if the domain of f is w^*-K analytic the result remains true for Banach spaces without copy of l^1 . The last hypothesis is necessary since the answer is not on $l^\infty = (l^1)^*$, [9].

All former results have been obtained in joint works with F. Delbaen [4,5,6,7], and are strongly connected with previous research by R.C.James [10], Gilles Godefroy [8,9] and the author [11], as well as on strong collaboration with Ruiz Galán, Cascales and Pérez [1,2,3,11].

[1] J. Orihuela and M. Ruiz Galán , *A coercive and nonlinear James's weak compactness theorem*, Nonlinear Analysis **75** (2012) 598–611.

[2] B. Cascales, J. Orihuela and M. Ruiz Galán, *Compactness, Optimality and Risk*, Computational and Analytical Mathematics. Chapter 10, 2013, 153–208. Springer.

[3] B. Cascales, J. Orihuela and A. Pérez, *One-sided James Compactness Theorem.* J. Math. Anal. Appli. **445**, Issue 2, 1267–1283 (2017).

[4] F. Delbaen and J. Orihuela, *Mackey constraints for James's compactness theorem and risk measures.* Journal of Mathematical Analysis and Applications 2019. 485(1):123764.

[5] F. Delbaen and J. Orihuela, *On the range of subdifferential in non reflexive Banach spaces.* J. Functional Analysis 2021.281(2):108915.

[6] F. Delbaen and J. Orihuela, *A multiset version of James's Theorem.* J. Functional Analysis 2022.

[7] F. Delbaen and J. Orihuela, *A nonlinear James's w^*-compactness theorem.* Work in progress 2024.

[8] G. Godefroy, *Boundaries of a convex set and interpolation sets.* Math. Annalen, **277** (1987), 173–184.

[9] G. Godefroy, *Five lectures in geometry of Banach spaces.* Seminar on Functional Analysis 1987. Notas de Matemática, Vol1, edited by J. Orihuela and A. Pallarés. Murcia University 1988.

[10] R.C. James. *Weakly compact sets*, Trans. Amer. Math. Soc. **113** (1964), 129-140.

[11] J. Orihuela, *Conic James' Compactness Theorem.* Journal of Convex Analysis (2018)(3), 1335–1344.

Trimming the Johnson bonsai

Friday
12.45-13.20

Jesús Castillo
Universidad de Extremadura

There are things that everybody knows even if nobody knows how they know them. For instance: *For $p > 1$, every complemented subspace of $\ell_p(\Gamma)$ is isomorphic to $\ell_p(I)$ for some $I \subset \Gamma$* is a folklore result that can be found in any Banach space book. The corresponding assertion for $p = 1$: *Every complemented subspace of $\ell_1(\Gamma)$ is isomorphic to $\ell_1(I)$ for some $I \subset \Gamma$* is folklore if we talk about Γ countable. However, the general proof for Γ uncountable is in no easily accessible place: a german paper of Köthe. Other proofs by Ortyńsk, Rosenthal, Rodríguez Salinas, Finol and Wojtowicz came later. Among them, Ortyńsky's proof stands out because it works the $p < 1$ case to obtain: *Given $0 < p < 1$, every complemented subspace of $\ell_p(\Gamma)$ is isomorphic $\ell_p(I)$ for some $I \subset \Gamma$*

The funny thing is that the proof works for different reasons depending on whether $p \leq 1$ or $p > 1$; and it is so because the different structural properties of $\ell_p(\Gamma)$ depending on whether $p < 1$, $p = 1$ or $p > 1$. Thus, for instance, what is true for $p = 1$ (every closed subspace of ℓ_1 contains a subspace isomorphic to ℓ_1 and complemented in ℓ_1) is false for $p < 1$. We will go to the common root of those results, which involves techniques of Johnson-Zippin and Moreno-Plichko,

- ($p > 1$). Every closed subspace of $\ell_p(\Gamma)$ with density character $\aleph > \aleph_0$ is an ℓ_p-vector sum $\ell_p(\aleph, H_m)$ of subspaces $H_m \subset \ell_p$
- ($p = \infty$). Every closed subspace of $c_0(\Gamma)$ with density character $\aleph > \aleph_0$ is a c_0-vector sum $c_0(\aleph, H_m)$ of subspaces $H_m \subset c_0$

and show that they are strongly false for $p \leq 1$: there are closed subspaces of $\ell_p(\Gamma)$ not even isomorphic to any ℓ_p-vector sum $\ell_p(\aleph, H_m)$ of subspaces $H_m \subset \ell_p$.

This talk is joint work with Félix Cabello and Yolanda Moreno.

Short talks

Algebraic structures of non-norm-attaining operators

Monday
16.15-16.35

Daniel L. Rodríguez-Vidanes
Technical University of Madrid

We will investigate the existence of large vector spaces within well-known families of continuous linear operators between Banach spaces that cannot be approximated by norm-attaining operators. Our analysis will include Lindenstrauss' famous counterexample to Bishop and Phelps' question regarding the density of norm-attaining operators, as well as Gowers' negative result concerning Lindenstrauss' property B of ℓ_p spaces $(1 < p < \infty)$.

We will also present generalizations and other significant results within the context of norm-attaining theory.

This talk is based on a joint work with S. Dantas, J. Falcó and M. Jung [1].

[1] S. Dantas, J. Falcó, M. Jung and D.L. Rodríguez-Vidanes. *Linear structures in the set of non-norm-attaining operators on Banach spaces.* Preprint arXiv:2311.17426 (2024).

Complemented copies of separable $\mathcal{C}(K)$ spaces in Banach spaces

Monday
16.40-17.00

Jakub Rondoš
University of Vienna

While the complementability of c_0 in Banach spaces (in particular, in $\mathcal{C}(K)$ spaces) has been studied quite extensively, there seems to be very little known about when a Banach space contains complemented copies of other separable $\mathcal{C}(L)$ spaces. In the talk, we speak about a joint work in progress in this direction with Damian Sobota. More concretely, let L be a metrizable compact space and E be a Banach space. We present a characterization of the presence of a complemented copy of $\mathcal{C}(L)$ in E in terms of the existence of a certain tree in $E \times E^*$. We also show how this can be applied to deduce that $\mathcal{C}(L)$ is complemented in $\mathcal{C}(K)$ for certain nonmetrizable compacta K.

Complemented subspaces of spaces $C(K \times L)$

Monday
17.05-17.25

Damian Sobota
University of Vienna

The classical theorem of Cembranos and Freniche asserts that, for every infinite compact spaces K and L, the Banach space $C(K \times L)$ of continuous real-valued functions on the product $K \times L$ contains a complemented copy of the Banach space c_0. During my talk I will present a significant extension of this result: If K and L map continuously onto a compact group G, then $C(K \times L)$ contains a complemented copy of $C(G)$. Consequently, if K and L are both non-scattered, then $C(K \times L)$ contains a complemented copy of $C(M)$ for any metric compact space M. Moreover, it follows that $C(\beta\mathbb{N} \times \beta\mathbb{N})$ contains a complemented copy of $C(M^\kappa)$ for any metric compact space M and cardinal number $1 \leq \kappa \leq 2^{\aleph_0}$. This is a joint work with Grzegorz Plebanek and Jakub Rondoš.

Descriptive complexity of the diameter two property

Tuesday
12.05-12.25

Esteban Martínez Vañó
Universidad de Granada

We compute the Borel complexity of the class of separable Banach spaces that satisfy the diameter two property. Furthermore, we show that the obtained complexity bound is optimal.

[1] López-Pérez, Ginés, Esteban Martínez Vañó and Abraham Rueda Zoca. *Computing Borel complexity of some geometrical properties in Banach spaces.* Preprint arXiv:2404.19457 (2024).

Daugavet and Δ-points

Tuesday
12.35-12.55

Triinu Veeorg
University of Tartu

A norm one element x of a Banach space is a *Daugavet point* (respectively, a Δ-*point*) if every slice of the unit ball (respectively, every slice of the unit ball containing x) contains an element that is almost at distance 2 from x. In this talk we take a look at some results on the topic. In particular, we examine the relation between a space containing a Δ-point and its dual containing a weak* Δ-point. In addition, we study how to renorm a Banach space with a Δ-point. The talk is based on [1].

[1] T. A. Abrahamsen, R. Aliaga, V. Lima, A. Martiny, Y. Perreau, A. Prochazka, and Triinu Veeorg, *Delta-points and their implications for the geometry of Banach spaces*, J. Lond. Math. Soc., II. Ser. **109** (2024), no. 5.

The Banach-Saks rank of a separable weakly compact set

Tuesday
13.05-13.25

Víctor Olmos Prieto
UNED

A well-known result by S. Banach and S. Saks states that every bounded sequence in $L^p([0,1])$, $1 < p < \infty$, has a Cesàro convergent subsequence. By taking the average of the subsequence multiple times, and through a recursive procedure, one can define a transfinite sequence of properties, thus giving rise to an ordinal rank or index for certain subsets of Banach spaces. We explore a different approach to this rank in the separable case using uniform families of finite subsets of integers. This allows us to characterize the existence of such a rank and study some of its properties from the point of view of Descriptive Set Theory.

Metric charaterization of Dunford-Pettis Lipschitz operators

Tuesday
15.30-15.50

Andrés Quilis

Laboratoire de Mathématiques de Besançon

Given a Lipschitz map $f\colon M \to N$ between metric spaces, there exists a linear operator $\hat{f}\colon \mathcal{F}(M) \to \mathcal{F}(N)$ between the respective Lipschitz-free spaces which extends f. In this talk, we will give a metric characterization for the maps f for which $\hat{f}$ is Dunford-Pettis, that is, transforms weakly null sequences into norm null sequences.

Special Ultrafilters, L-orthogonality, and octahedral norms

Tuesday
15.55-16.15

Luis Sáenz

Univesidad Nacional Autónoma de México

One of the questions in the area of octahedral norms and L-orthogonal sequences is: given an L-orthogonal sequence $(x_n)_{n\in\mathbb{N}} \subseteq X$ can we assure there is an L-orthogonal element in its w^*-closure in X^{**}? Turns out this question is independent of the usual axioms of set theory and the existence of Q-points, a special type of ultrafilter, decides it. I will present an overview of the area, the important set theoretic concepts, and a sketch of the proof of this latter theorem.

Generic nonexpansive Hilbert space mappings

Tuesday
16.20-16.40

Davide Ravasini

Universität Leipzig

We consider a closed, convex set C in an infinite-dimensional, separable Hilbert space H and endow the space $\mathcal{N}(C)$ of all nonexpansive mappings $f\colon C \to C$ with the topology of pointwise convergence. We introduce the definition of *somewhat bounded set* and connect this property with the generic existence of fixed points, in the sense of Baire categories. Namely, if C is somewhat bounded, then a generic $f \in \mathcal{N}(C)$ admits a fixed point, whereas if C is not somewhat bounded, then a generic $f \in \mathcal{N}(C)$ does not have any fixed points. This results in a topological 0–1 law: the set of all $f \in \mathcal{N}(C)$ with a fixed point is either meagre or residual. We further discuss the generic uniqueness of such fixed points and the convergence of the iterates of a generic $f \in \mathcal{N}(C)$ to a fixed point of f.

A new presentation of Pełczyński's universal space

Wednesday
12.50-13.10

Barnabás Farkas
TU Wien

In his paper *Universal bases* (1969) Pełczyński constructed a Banach space U with an unconditional basis such that U contains complemented copies of all such spaces, he also showed that U is unique up to isomorphism.

A new and simple construction of such a universal space $X_{\mathcal{P}}$ will be presented. Furthermore, answering a question posed by Pełczyński, we will see that, contrary to the canonical basis of U, in $X_{\mathcal{P}}$ the canonical basis itself is not universal.

This result was proved in the paper *The Zoo of combinatorial Banach spaces* (submitted) by P. Borodulin-Nadzieja, B. Farkas, S. Jachimek, A. Pelczar-Barwacz.

Ball separation characterization of small diameter properties

Wednesday
13.15-13.35

Sudeshna Basu
Loyola University

Ball separation property an important concept of the geometry of Banach space was first con- sidered by Mazur in [M]. Later, it was developed by Whitfield, Zizler [WZ], Sersouri [S], Chen, Lin [CL], Corson, Lindenstrauss [CL3], and Giles [G], to name a few. Small diameter properties of Banach spaces are related to the existence of dentability, huskability and small combination of slices of the closed unit ball of Banach spaces, for more details, see [BR] [B] and [BS]. In other words, Small diameter properties are localised versions of well-known geometric properties Radon-Nikodym Property, Point of Continuity Property and Strong Regularity to the closed unit ball of Banach spaces, for more details , see [GGMS]. In this work, we completely characterize Ball Dentable Property (BDP), Ball Huskable Property (BHP) and their w^*-versions in terms of ball separation properties in Banach spaces. We also obtain a necessary ball separation condition for any Banach space with Ball Small Combination of Slice Property (BSCSP). Next, we present the notion of semi PC, semi SCS point and their related w^*-versions that extend the notion of semi denting point and its w^*-version. We also establish their ball separation characterization. The ball separation characterization of semi w^*-PC leads us to give a new characterization of Property (II). We prove that a Banach space X has property (II) if and only if every point in S_{X^*} is w^*-semi PC of B_{X^*}. At the end of this note we introduce the notion of $\mathcal{A}$-Small Combination of Slice point and obtain a necessary ball separation condition for the existence of $\mathcal{A}$-Small Combination of Slice point.

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[BR] S. Basu, T. S. S. R. K. Rao, *On Small Combination of slices in Banach Spaces*, Extracta Mathematicae 31 (1) 1–10 (2016).

[BS] S. Basu, S. Seal, *Small combination of slices, dentability and stability results of small diameter properties in Banach spaces*, J. Math. Anal. Appl. 507 (2) (2022). Online access, https://doi.org/10.1016/j.jmaa.2021.125793

[CL] D. Chen, B. L. Lin, *Ball separation properties in Banach spaces*, Rocky Mountain J. Math. 28 (3) 835–873 (1998).

[CL3] H. H. Corson, J. Lindenstrauss, *On weakly compact subsets of Banach spaces*, Proc. Amer. Math. Soc. 17 (2) 407–412 (1966).

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On Lipschitz-free spaces isomorphic to ℓ_1

Thursday
12.45-13.05

Rubén Medina
Universidad de Granada

We study transportation cost spaces over finite metric spaces M having a basis C-equivalent to the ℓ_1 basis for $C \geq 1$.

A basis is called random if its canonical projections are random retractions, where a random retraction from a metric space M to $N \subset M$ is a linear projection $P : \mathcal{F}(M) \to \mathcal{F}(N)$ preserving probabilities.

In this paper, we find a metric characterization of the existence of a random basis in $\mathcal{F}(M)$ C-equivalent to the ℓ_1 basis in terms of the distortion between the original metric of M and certain probabilistic distributions over tree metrics of M.

M-ideals of compact operators and norm attaining operators

Thursday
13.10-13.30

Manwook Han
Chungbuk National University

A closed subspace J of a Banach space X is said to be an M-ideal if $X^* = J^* \oplus_1 J^\perp$. It is clear that if X is the ℓ_∞-sum of J and its complement, then J is an M-ideal. The converse is not true in general. Especially, the space $\mathcal{L}(X, Y)$ of linear operators cannot be represented by such ℓ_∞-sum with the space $\mathcal{K}(X, Y)$ of compact operators if $\mathcal{K}(X, Y)$ is proper. But $\mathcal{K}(X, Y)$ is an M-ideal for some pair of Banach spaces (X, Y). In this talk, we introduce various known properties that are satisfied in these cases and present new results that can be linked to the study on norm attaining operators.

Isometric Jordan isomorphisms of group algebras

Thursday
(Room 1)
15.30-15.50

Cristian Castillo Godoy
Universidad de Alicante

Let A and B be Banach algebras. An operator $\Phi\colon A \to B$ is said to be a *Jordan homomorphism* if

$$\Phi(a \circ b) = \Phi(a) \circ \Phi(b) \quad (a, b \in A),$$

where $\circ$ denotes the Jordan product ($a \circ b = ab + ba$).

Inspired by Kadison, who proved that each isometric Jordan isomorphism from a von Neumann algebra onto a C^*-algebra is the direct sum of an isometric isomorphism and an isometric anti-isomorphism, we show that each contractive Jordan isomorphism between group algebras is either an isometric isomorphism or an isometric anti-isomorphism.

We then use this result to give descriptions of two-sided zero product preservers and approximately local automorphisms between group algebras.

On some measures related to ideals of multilinear operators and interpolation

Thursday
(Room 2),
15.30-15.50

Antonio Manzano
Universidad de Burgos

We shall introduce some functionals or measures associated to ideals of multilinear operators that, under certain assumptions on the corresponding ideal, allow to characterize the operators that belong to the ideal as those having measure equal zero. These measures can be considered as an extension, in the setting of multilinear operators, of the outer measure or the inner measure (depending on the case) defined by Kari Astala and Hans-Olav Tylli, respectively, for an ideal of linear operators.

We shall also establish different interpolation formulas for these multilinear measures and, as a consequence of them, interpolation results involving ideals of multilinear operators.

The talk is based on joint works with Pilar Rueda (Universidad de Valencia, Spain) and Enrique A. Sánchez-Pérez (Universidad Politécnica de Valencia, Spain).

Fedorchuk compacta and LUR renormability

Thursday
(Room 1)
15.55-16.15

Todor Manev
Sofia University

Fedorchuk compact spaces are compact spaces that can be obtained as limits of inverse systems where the neighboring bonding mappings are fully closed and have metrizable fibers. A continuous onto mapping is called fully closed if the intersection of the images of any two closed disjoint sets is finite. We show that $C(X)$ admits an equivalent locally uniformly rotund norm whenever X is a Fedorchuk compact satisfying some additional conditions. We adapt the construction of Fedorchuk compacta to obtain some other classes of compact spaces as limits of inverse systems with fully closed neighboring bonding mappings.

This study is financed by the European Union-NextGenerationEU, through the National Recovery and Resilience Plan of the Republic of Bulgaria, project № BG-RRP-2.004-0008-C01.

Separating all diameter two properties in spaces of Lipschitz functions

Thursday
(Room 2)
15.55-16.15

Jaan Kristjan Kaasik
University of Tartu

We introduce diameter two properties in Banach spaces and study these properties in spaces of Lipschitz functions. In particular, we will show that all diameter two properties and their weak-star counterparts differ from each other in spaces of Lipschitz functions. This highlights that there exists an octahedral Lipschitz-free space whose bidual is not octahedral. The talk is based on recent preprints and new unpublised results.

Most Iterations of Projections Converge

Thursday
(Room 1)
16.25 - 16.45

Daylen Thimm
University of Innsbruck

Consider three closed linear subspaces C_1, C_2, and C_3 of a Hilbert space H and the orthogonal projections P_1, P_2 and P_3 to them. Halperin showed that a point in $C_1 \cap C_2 \cap C_3$ can be found by iteratively projecting any point $x_0 \in H$ onto all the sets in a periodic fashion. The limit point is then the projection of x_0 onto $C_1 \cap C_2 \cap C_3$. Nevertheless, a non-periodic projection order may lead to a non-convergent projection series, as shown by Kopecká, Müller, and Paszkiewicz. This raises the question how many projection orders in $\{1, 2, 3\}^{\mathbb{N}}$ are "well behaved" in the sense that they lead to a convergent projection series. De Brito, Melo, and da Cruz Neto showed that the "well behaved" projection orders form a large subset in the sense of measure, as they have full product measure. We show that also from a topological viewpoint the set of "well behaved" projection orders is a large subset: it contains a dense G_δ subset in the product topology.

Approximate Morse-Sard theorem in nonseparable Banach spaces

Thursday
(Room 2)
16.25 - 16.45

Miguel García Bravo
University Complutense of Madrid

The main purpose of this talk is to extend from the separable to the not necessarily separable situation, some previous results concerning what one can regard as an approximate strong version of the Morse-Sard theorem for mappings between Banach spaces E and F.

Contrary to what happens in finite dimensions, where one can dispose of the Morrse-Sard theorem, there exists functions $f : \ell_2(\mathbb{N}) \to \mathbb{R}$ of class C^∞ so that $f(C_f) = [0, 1]$, where $C_f = \{x \in \ell_2 : Df(x) = 0\}$ is the set of critical points. However, it was proven in 2004 by Azagra and Cepedello that any continuous function $f : \ell_2(\mathbb{N}) \to \mathbb{R}$ can be uniformly approximated by C^∞ functions without critical points. In this presentation we will see that one gets the same type of approximation result for continuous functions $f : \ell_2(\Gamma) \to \mathbb{R}$, where Γ is an arbitrary infinite set. And similar results are also true for more general nonseparable Banach spaces E and F and continuous functions $f : E \to F$.

This is a joint work with Daniel Azagra and Mar Jiménez-Sevilla.

Linear Chaos in Lipschitz-free spaces

Thursday
(Room 1)
16.50-17.10

Christian Cobollo
Universidad Politécnica de Valencia

Along this short talk, we will summarise and present some new results regarding the relation between a non-linear dynamical system of a Lipschitz self-mapping f on a metric space M and its linear counterpart $T_f \in \mathcal{L}(\mathcal{F}(M))$ (the linearization operator of f) on the Lipschitz-free space. We will provide results, examples and counterexamples surrounding well-known properties in the field of Linear Dynamics, like different versions of transitivity, hypercyclicity, Disjoint-transitiveness, or the operator specification property.

Slicely countably determined points in Banach spaces

Thursday
(Room 1)
17.15-17.35

Marcus Lõo
University of Tartu

In 2010, A. Avilés, V. Kadets, M. Martín, J. Merí, and V. Shepelska introduced a (non-trivial) class of Banach spaces which contains both separable spaces with the RNP and separable Asplund spaces. In this talk, we will be discussing SCD points of a bounded and convex subset of a Banach space, which extends the notions of denting points, strongly regular points, and, furthermore, allows us to investigate the SCD phenomena in the non-separable case. We present some properties of SCD points, provide stability results and (either fully or partly) characterize these points in L_1-preduals, Lipschitz-free spaces and projective tensor products. Finally, we discuss some applications of SCD points.

On the complete separation of unique ℓ_1 spreading models and the Lebesgue property

Harrison Gaebler
University of North Texas

A Banach space X is said to have the *Lebesgue property* (LP) if every Riemann-integrable function $f : [0,1] \to X$ is Lebesgue almost-everywhere continuous. In the recent paper [2], the LP is characterized in terms of a new sequential asymptotic structure such that for the statements:

(S1) every asymptotic model of X is equivalent to the unit vector basis of ℓ_1

(S2) X has the LP

(S3) every spreading model of X is equivalent to the unit vector basis of ℓ_1

the implications (S1) $\implies$ (S2) and (S2) $\implies$ (S3) are true. On the other hand, it is also shown in [**?**] that neither converse implication is true and one can therefore ask the following two questions regarding what is known as the *complete separation* of these asymptotic structures:

(Q1) Does there exist a subspace Y of X such that every asymptotic model of Y is equivalent to the unit vector basis of ℓ_1 if X has the LP?

(Q2) Does there exist a subspace Y of X such that Y has the LP if every spreading model of X is equivalent to the unit vector basis of ℓ_1?

It is proved in [2] that the Banach space X_{iw} due to Argyros and Motakis in [1] has the LP and this answers (Q1) in the negative because every subspace of X_{iw} contains an asymptotic model that is equivalent to the unit vector basis of c_0. However, (Q2) is not addressed in [2]. In this short talk based on joint work with Bunyamin Sari and Pavlos Motakis, I will show why X_{iw} has the LP and, in so doing, motivate the similar construction in [3] of a new Banach space $X_{\mathcal{D}}$ that answers (Q2) in the negative.

[1] S. Argyros and P. Motakis. On the complete separation of asymptotic structures in Banach spaces. *Adv. Math.*, 362, 2020.

[2] H. Gaebler and B. Sari. Banach spaces with the Lebesgue property of Riemann integrability. *J. Funct. Anal.*, 287 (2), 2024.

[3] H. Gaebler, P. Motakis, and B. Sari. On the complete separation of unique ℓ_1 spreading models and the Lebesgue property of Banach spaces. 2024. arXiv:math/2402:14687.

Posters

A note on the Bishop–Phelps–Bollobás theorem

Jaagup Kirme
Tartu University

The Bishop–Phelps theorem states that for any Banach space X the set of norm-attaining functionals is dense in the dual $X^* = \mathcal{L}(X, \mathbb{K})$. The Bishop–Phelps–Bollobás theorem sharpens this result. If we instead view (norm-attaining) operators between two banach spaces X and Y, such a general theorem will not hold. In the presentation we will define this generalisation of the BP and BPB theorems and bring a positive example of one such pair of Banach spaces where the theorems hold.
