UNIVERSITAT POLITÈCNICA DE VALÈNCIA

Linear Chaos in Lipschitz-free spaces

Based on a joint work with Alfred Peris

Christian Cobollo

New Perspectives in Banach Spaces and Banach Lattices
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Lipschitz-free Preliminaries

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The ***Lipschitz-free space*** of M is defined as

$$\mathcal{F}(M) := \overline{\text{span}}\{\delta_x : x \in M\} \ \ (\subset \mathsf{Lip}_0(M)^*),$$

and it is indeed a predual of $\mathsf{Lip}_0(M)$.

Lipschitz-free Preliminaries

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Roughly speaking, we can think of $\mathcal{F}(M)$ as taking M and providing it with a linear structure in which distinct (non-zero) points in M are now linearly independent,and the endowed norm is the one keeping the original metric structure of M.Thus, $\|\delta_x\| = d(x,0)$, or more generally, $\|\delta_x - \delta_y\| = d(x,y)$.

Lipschitz-free Preliminaries

Why is $\mathcal{F}(M)$ so special?

The Universal Property (<u>Linearization</u>)

Lipschitz-free Preliminaries

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The Universal Property (Linearization)

$$f \in \mathrm{Lip}_0(M, X)$$

$$\begin{tikzcd}
M \arrow[r, "f"] \arrow[d, "\delta"'] & X \\
\mathcal{F}(M) \arrow[ur, Rightarrow, "T_f"'] &
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- Andrés' talk (Tuesday)!

Linear Dynamics

The properties we like

Definition

f is **hypercyclic** if:
$\exists x \in M \ : \ \mathrm{Orb}\,(x, f)$ is dense.

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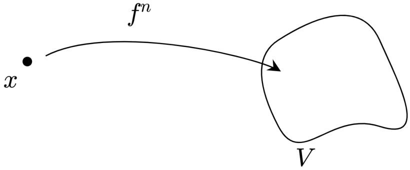

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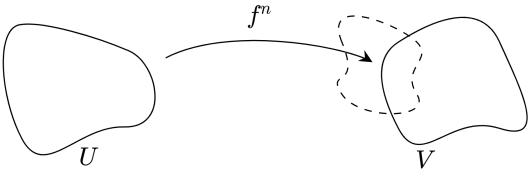

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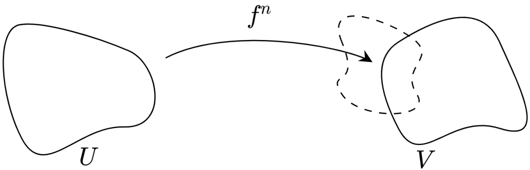
f is **mixing** if:
For $U, V \subset M$ open, $\exists n_0 \in \mathbb{N}\ : f^n(U) \cap V \neq \emptyset$, $\forall \boldsymbol{n} \geq \boldsymbol{n_0}$



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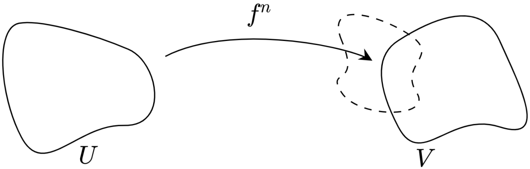

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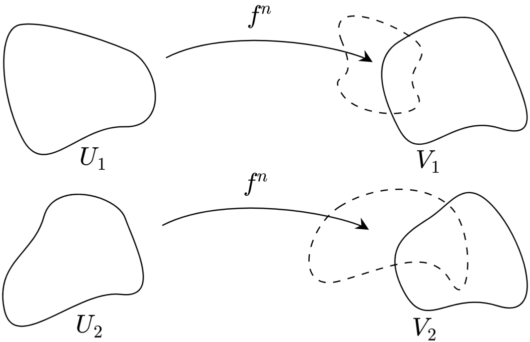
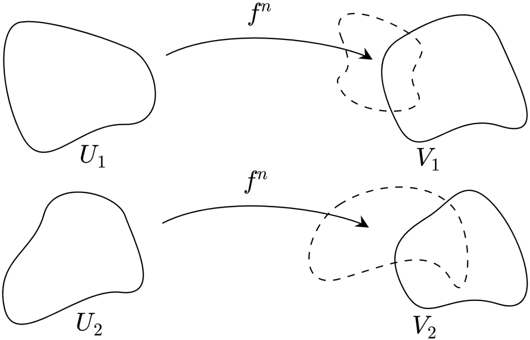


Linear Dynamics

The properties we like

Definition

f is **weakly mixing** if:
For $U_1, U_2, V_1, V_2 \subset M$ open, $\exists n \in \mathbb{N}\ : f^n(U_1) \cap V_1 \neq \emptyset$, $f^n(U_2) \cap V_2 \neq \emptyset$



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$$\text{mixing} \implies \text{weakly mixing} \implies \text{transitive}$$

Linear Dynamics in $\mathcal{F}(M)$

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Theorem (A. Peris, M. Murillo-Arcila, 2015)

Let $T \in \mathcal{L}(X)$ and $K \subset X$ a T-invariant set, $0 \in K$, $\overline{\mathrm{span}(K)} = X$. If $T_{|K}$ is weakly mixing, weakly mixing and chaotic, or mixing, then T has the same property.

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Example: Let $T_f \in \mathcal{L}(\mathcal{F}(M))$, $\delta(M) \in \mathcal{F}(M)$ is T_f-invariant, $0 \in \delta(M)$, and $\overline{\mathrm{span}(\delta(M))} = \mathcal{F}(M)$. But, $T_f(\delta_x) = \delta_{f(x)}$.

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Corollary

If $f \in \mathrm{Lip}_0(M, M)$ is weakly mixing, weakly mixing and chaotic, or mixing, then $T_f \in \mathcal{L}(\mathcal{F}(M))$ has the same property.

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Proposition (Abbar, Coine, Petitjean, 2021)

Let M be a metric space with non-isolated $0 \in M$, and $f \in \mathrm{Lip}_0(M, M)$. TFAE:

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Proof.

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Idea: Dense in a linearly dense $\implies$ linearly dense. □

Linear Dynamics in $\mathcal{F}(M)$

We will see—in a quite more general set-up— that every “strong enough” property is inherited from M to $\mathcal{F}(M)$.

Linear Dynamics in $\mathcal{F}(M)$

A way to define your own transitivities!

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If, moreover,

(iii) If $A, B \in \mathcal{A}$ then $A \cap B \in \mathcal{A}$,

then we will say that $\mathcal{A}$ is a **filter**.

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Definition

Let f be a map in M, $\mathcal{A} \subset \mathcal{P}(\mathbb{N})$ a Furstenberg family.We say that f is $\mathcal{A}$-transitive if for every U, V open sets,

$$N_f(U, V) \in \mathcal{A}.$$

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Example (Furstenberg families)

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We can even work with several maps at once!

We may even work with more than one map!

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Notation

If we are considering several maps $f_1, ..., f_N$ on M, and open sets $U_1, ..., U_N \subset M$,

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Notation

If we are considering several maps $f_1, ..., f_N$ on M, and open sets $U_1, ..., U_N \subset M$, (please) allow us to denote

$$f := f_1 \times f_2 \times ... \times f_N$$
$$U := U_1 \times ... \times U_N$$

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Definition

$f_1, ..., f_N$ are **disjoint transitive** if for every $U_0, U_1, ..., U_N \subset M$ open,

$$d\text{-}\mathcal{N}_f(U_0, U) := \{n \in \mathbb{N} : U_0 \cap \bigcap_{i=1}^{N} f_i^{-n}(U_i) \neq \emptyset\} \neq \emptyset$$

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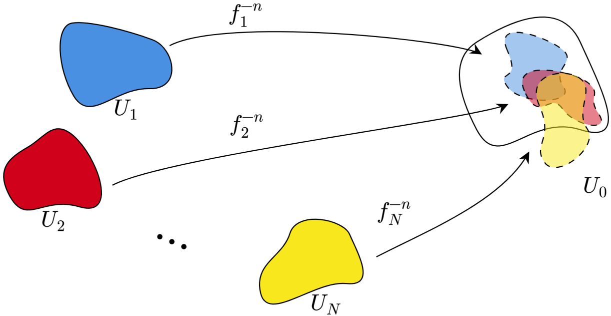

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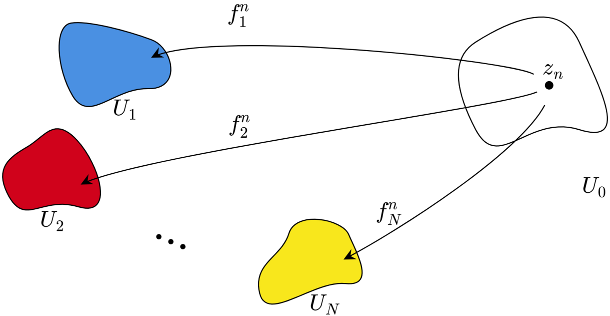

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Let M (no isolated points) and $f_1, ..., f_N$. Then, $d\text{-}\mathcal{N}_f(U_0, U) \neq \emptyset$ if and only if $d\text{-}\mathcal{N}_f(U_0, U)$ is infinite.

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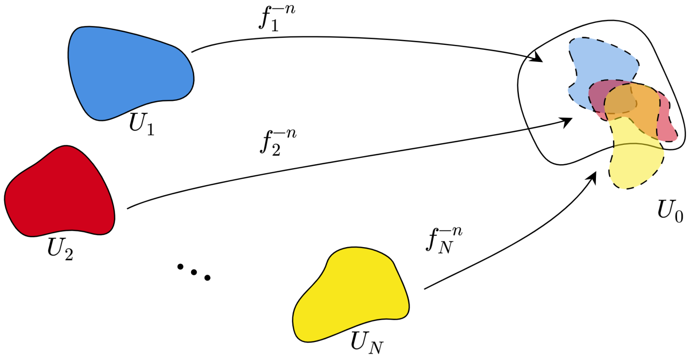

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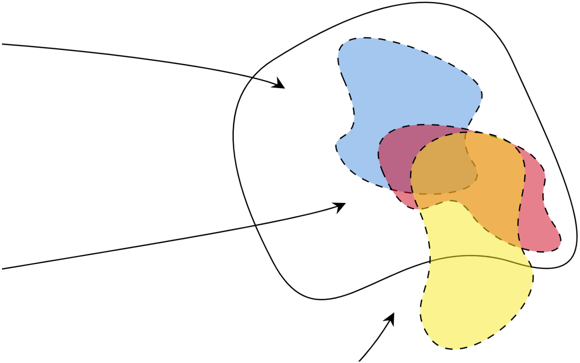

Linear Dynamics in $\mathcal{F}(M)$

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Let M (no isolated points) and $f_1, ..., f_N$. Then, d-$\mathcal{N}_f(U_0, U) \neq \emptyset$ if and only if d-$\mathcal{N}_f(U_0, U)$ is infinite.

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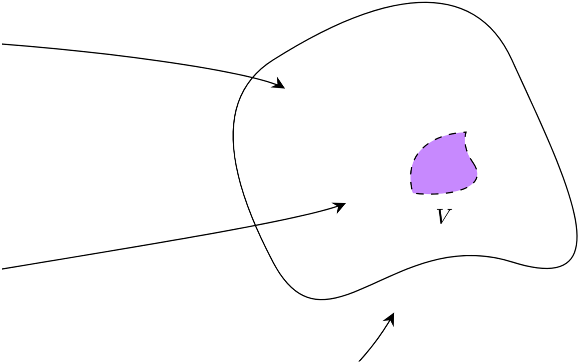

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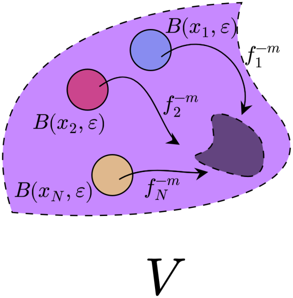

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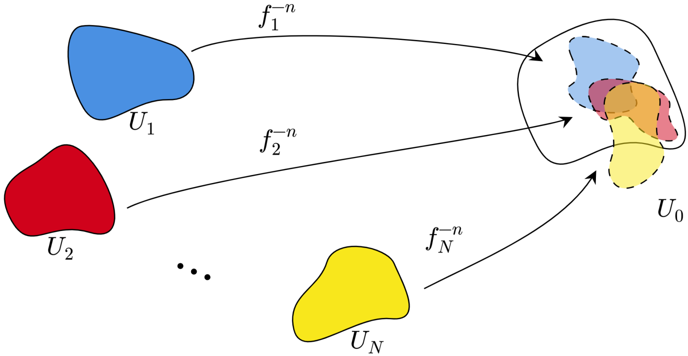

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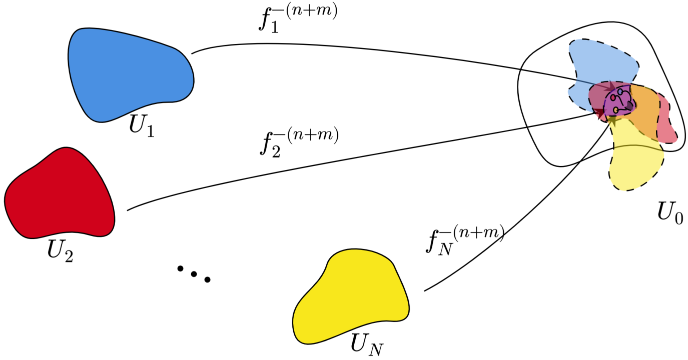

Linear Dynamics in $\mathcal{F}(M)$

Definition

Let $f_1, ..., f_N$ maps defined in M, $\mathcal{A} \subset \mathcal{P}(\mathbb{N})$ be a Furstenberg family. We say that $f_1, ..., f_N$ are **disjoint $\mathcal{A}$-transitive** if for every non-empty open subsets $U_0, U_1, ..., U_N \subset M$

$$d\text{-}\mathcal{N}_f(U_0, U) \in \mathcal{A}.$$

Linear Dynamics in $\mathcal{F}(M)$

Inheritance of strong properties

Theorem (Ch. C. and A. Peris, 2024+)

Let $f_1, ..., f_N \in \text{Lip}_0(M, M)$ disjoint $\mathcal{A}$-transitive maps in M, with $\mathcal{A}$ being a filter. Then its corresponding operators $T_{f_1}, ..., T_{f_N} \in \mathcal{L}(\mathcal{F}(M))$ are disjoint $\mathcal{A}$-transitive.

Linear Dynamics in $\mathcal{F}(M)$

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Linear Dynamics in $\mathcal{F}(M)$

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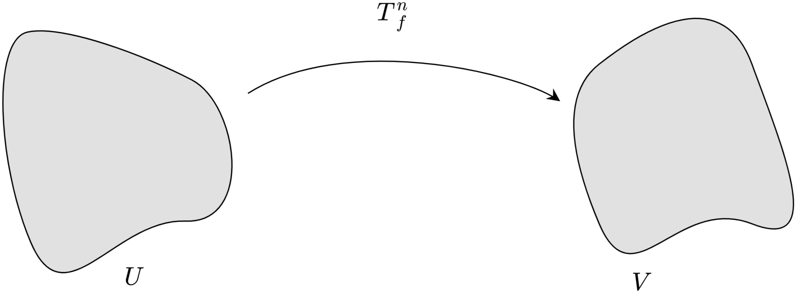

Linear Dynamics in $\mathcal{F}(M)$

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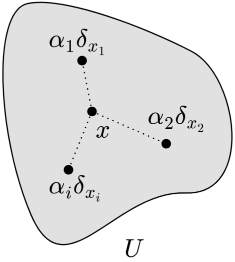

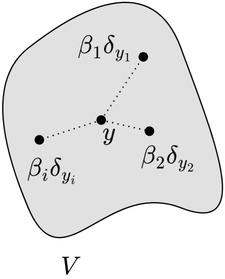

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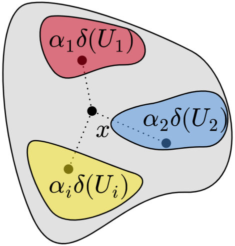

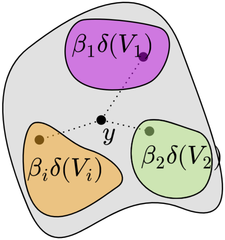

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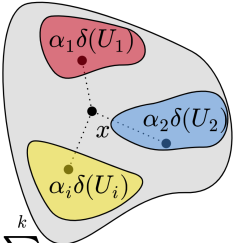
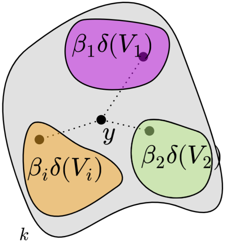


$$\sum_{i=1}^{k} \alpha_i\delta(U_i) \subset U$$



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Linear Dynamics in $\mathcal{F}(M)$

Inheritance of strong properties

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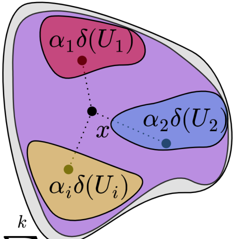

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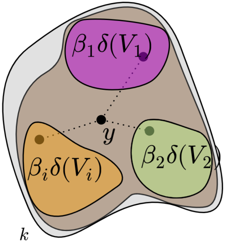

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Linear Dynamics in $\mathcal{F}(M)$

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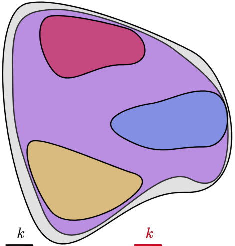
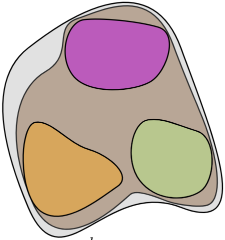


$$\sum_{i=1}^{k}\alpha_i\delta(U_i)+\sum_{i=1}^{k}\beta_i\delta(W_0)\subset U$$



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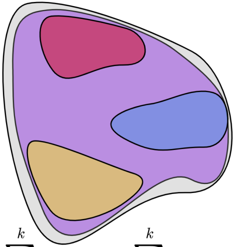

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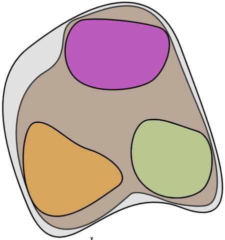

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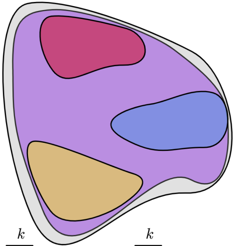
Linear Dynamics in $\mathcal{F}(M)$

Inheritance of strong properties

Idea:

$$\begin{tikzcd} U_i \arrow[r, bend left, "f"] & W_0 \end{tikzcd}$$



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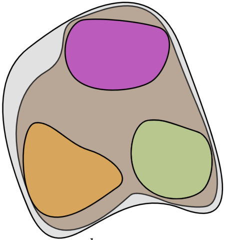

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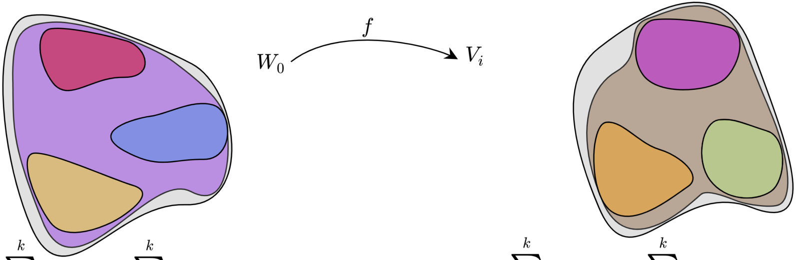
$$W_0 \xrightarrow{f} V_i$$

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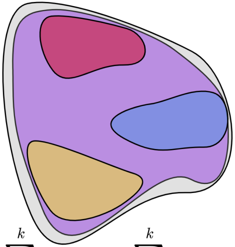

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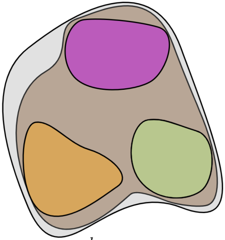

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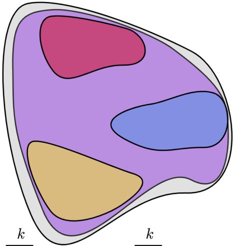

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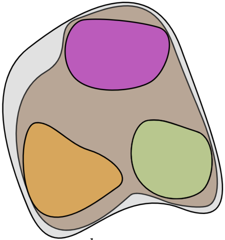

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Linear Dynamics in $\mathcal{F}(M)$

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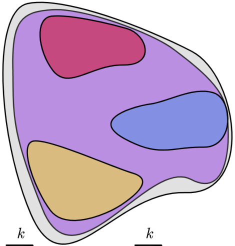
Idea:

$$\bigcap_{i=1}^{k} (N_f(U_i, W_0) \cap N_f(W_0, V_i))$$



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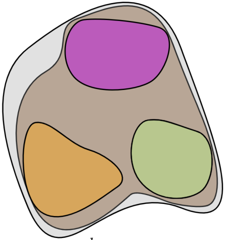

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Linear Dynamics in $\mathcal{F}(M)$

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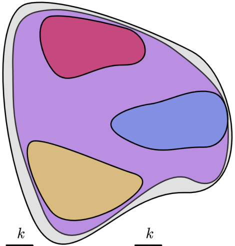
Idea:

$$\bigcap_{i=1}^{k} (N_f(U_i, W_0) \cap N_f(W_0, V_i)) \in \mathcal{A}$$



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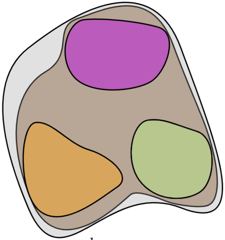

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Linear Dynamics in $\mathcal{F}(M)$

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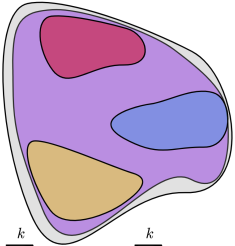
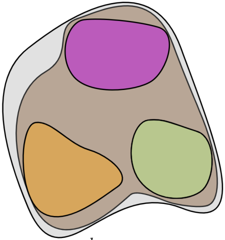
Idea:

$$N_{T_f}(U,V) \supset \bigcap_{i=1}^{k} \left(N_f(U_i,W_0) \cap N_f(W_0,V_i)\right) \in \mathcal{A}$$





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Linear Dynamics in $\mathcal{F}(M)$

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Idea:

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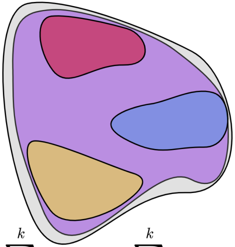

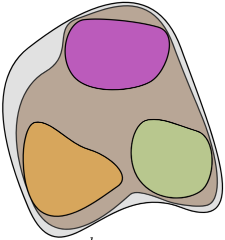

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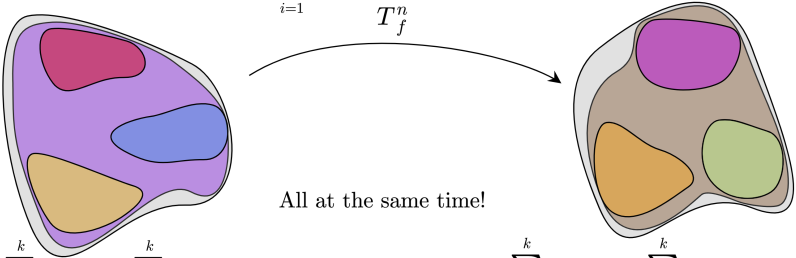

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Theorem (Ch. C. and A. Peris, 2024+)

Let $f_1, ..., f_N \in \mathrm{Lip}_0(M, M)$ disjoint $\mathcal{A}$-transitive maps in M, with $\mathcal{A}$ being a filter. Then its corresponding operators $T_{f_1}, ..., T_{f_N} \in \mathcal{L}(\mathcal{F}(M))$ are disjoint $\mathcal{A}$-transitive.

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Corollary (Ch. C. and A. Peris, 2024+)

Let $f \in \text{Lip}_0(M, M)$ $\mathcal{A}$-transitive map in M, with $\mathcal{A}$ being a filter. Then its corresponding operator $T_f \in \mathcal{L}(\mathcal{F}(M))$ is $\mathcal{A}$-transitive.

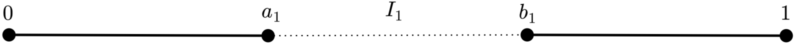
Linear Dynamics in $\mathcal{F}(M)$

Backward Shift on the Cantor set

Linear Dynamics in $\mathcal{F}(M)$

Backward Shift on the Cantor set

Let $\mathfrak{C}$ be the middle third ternary Cantor set.



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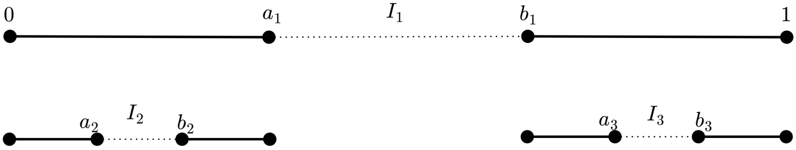

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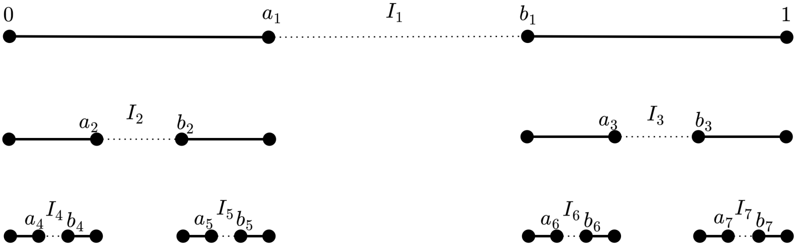

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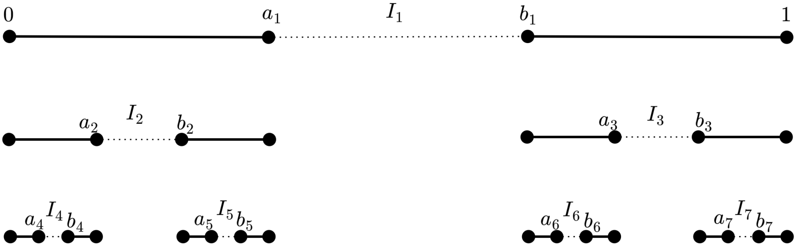

Thus, $\mathfrak{C} = [0,1] \backslash \left(\bigcup_{n=1}^{\infty} I_n \right)$.

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Linear Dynamics in $\mathcal{F}(M)$

Backward Shift on the Cantor set

Thus, $\mathfrak{C} = [0,1]\backslash\big(\bigcup_{n=1}^{\infty} I_n\big)$.

$$(d(b_n, a_n))_{n=1}^{\infty} = \Big(\frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^2}, \frac{1}{3^3}, \frac{1}{3^3}, \frac{1}{3^3}, \frac{1}{3^3}, \ldots\Big),$$

Linear Dynamics in $\mathcal{F}(M)$

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Every $t \in \mathfrak{C}$ has a ternary representation as a sequence of 0's and 2's, $s(t) := (s_1(t), s_2(t), ...)$ such that $t = \sum \frac{s_n(t)}{3^n}$ (where $s_n(t) \in \{0, 2\}$).

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Now, consider the (Lipschitz) map $\sigma \colon \mathfrak{C} \to \mathfrak{C}$ known as the ***backward shift***, defined by the expression

$$\sigma(t) := \sum \frac{s_{n+1}(t)}{3^n}.$$

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Backward Shift on the Cantor set

Thus, $\mathfrak{C} = [0,1] \backslash \big(\bigcup_{n=1}^{\infty} I_n \big)$.

$$(d(b_n, a_n))_{n=1}^{\infty} = \Big(\frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^2}, \frac{1}{3^3}, \frac{1}{3^3}, \frac{1}{3^3}, \frac{1}{3^3}, \ldots \Big),$$

Every $t \in \mathfrak{C}$ has a ternary representation as a sequence of 0's and 2's, $s(t) := (s_1(t), s_2(t), \ldots)$ such that $t = \sum \frac{s_n(t)}{3^n}$ (where $s_n(t) \in \{0, 2\}$).

Now, consider the (Lipschitz) map $\sigma \colon \mathfrak{C} \to \mathfrak{C}$ known as the ***backward shift***, defined by the expression

$$\sigma(t) := \sum \frac{s_{n+1}(t)}{3^n}.$$

$$(s_1(t), s_2(t), s_3(t), \ldots) \xrightarrow{\sigma} (s_2(t), s_3(t), \ldots)$$

Linear Dynamics in $\mathcal{F}(M)$

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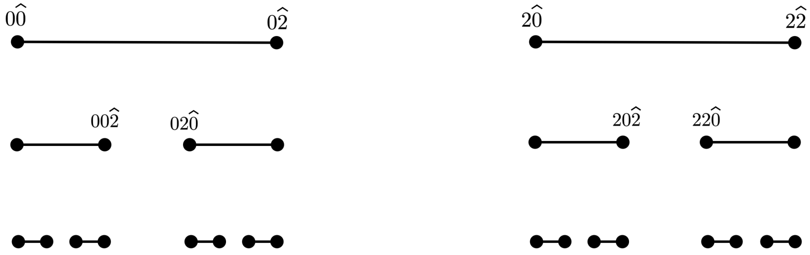
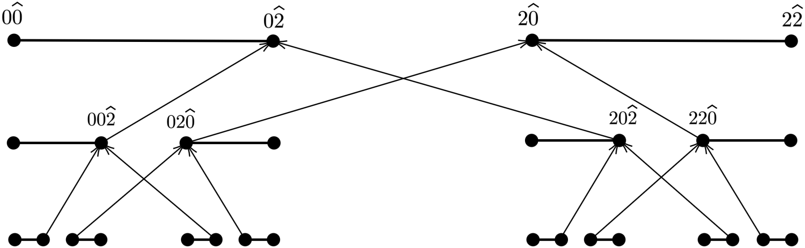


Linear Dynamics in $\mathcal{F}(M)$

Backward Shift on the Cantor set



Linear Dynamics in $\mathcal{F}(M)$

Backward Shift on the Cantor set

Proposition (Ch. C. and A. Peris, 2024+)

The operator $T_\sigma \in \mathcal{L}(\mathcal{F}(\mathfrak{C}))$ is conjugated to $S_\sigma \in \mathcal{L}(\ell_1)$ such that $S_\sigma(e_n) = 3e_{[n/2]}$ for $n \geq 2$, and $S_\sigma(e_1) = -3\sum_{n=1}^{\infty} d(b_n, a_n)e_n$ (which, moreover, is a fixed point).

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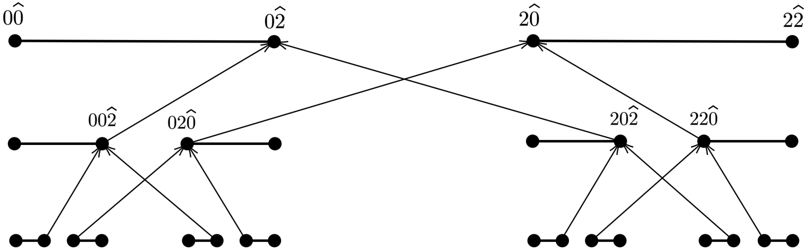
$$M_\sigma = 3\begin{bmatrix} -d_1 & 1 & 1 & 0 & 0 & 0 & 0 & \cdots \\ -d_2 & 0 & 0 & 1 & 1 & 0 & 0 & \cdots \\ -d_3 & 0 & 0 & 0 & 0 & 1 & 1 & \cdots \\ \vdots & \vdots & & & \ddots & \ddots & & \end{bmatrix}$$

Linear Dynamics in $\mathcal{F}(M)$

Backward Shift on the Cantor set



Linear Dynamics in $\mathcal{F}(M)$

Backward Shift on the Cantor set

Proposition (Ch. C. and A. Peris, 2024$+$)

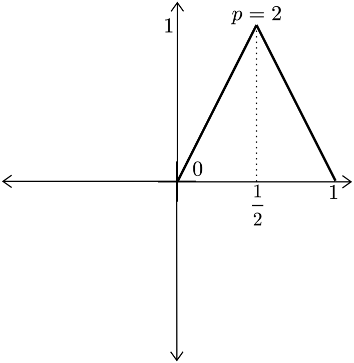
Let $\sigma : \mathfrak{C} \to \mathfrak{C}$ be the backward shift map in the usual ternary Cantor set. Therefore $T_{\sigma^{m_1}}, ..., T_{\sigma^{m_N}}$ are disjoint mixing operators in $\mathcal{F}(\mathfrak{C})$.

Linear Dynamics in $\mathcal{F}(M)$

"What if I'm not strong?" The anti-symmetric tent map

Linear Dynamics in $\mathcal{F}(M)$

“What if I’m not strong?” The anti-symmetric tent map



Linear Dynamics in $\mathcal{F}(M)$

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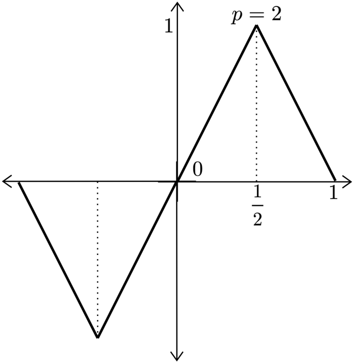

Linear Dynamics in $\mathcal{F}(M)$

"What if I'm not strong?" The anti-symmetric tent map

Proposition (Ch. C. and A. Peris, 2024$+$)

Let f be the anti-simmetric tent map in $[-1,1]$. Therefore $T_{f^{m_1}}, ..., T_{f^{m_N}}$ are disjoint mixing operators in $\mathcal{F}([-1,1])$.

Linear Dynamics in $\mathcal{F}(M)$

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Let f be the anti-simmetric tent map in $[-1,1]$. Therefore $T_{f^{m_1}},...,T_{f^{m_N}}$ are disjoint mixing operators in $\mathcal{F}([-1,1])$. Although f (and no power of f) is even transitive!

Linear Dynamics in $\mathcal{F}(M)$

Lipschitz Disjoint Hypercyclicity Criterion

Theorem (**Lipschitz Disjoint Hypercyclicity Criterion**) (Ch. C., A. Peris, 2024+)

Let $f_1, f_2, ..., f_N \in \mathrm{Lip}_0(M, M)$

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Linear Dynamics in $\mathcal{F}(M)$

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Linear Dynamics in $\mathcal{F}(M)$

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Linear Dynamics in $\mathcal{F}(M)$

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Linear Dynamics in $\mathcal{F}(M)$

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2. $g_{i,k}(x_i) \xrightarrow{k\to\infty} 0$ for every $x_i \in M_i$.
3. $(f_i^{n_k} \circ g_{j,k})(x_i) \xrightarrow{k\to\infty} \begin{cases} x_i & \text{if } j = i; \\ 0 & \text{if } j \neq i; \end{cases}$ for every $x_i \in M_i$.

Linear Dynamics in $\mathcal{F}(M)$

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Then $T_{f_1}, ..., T_{f_N}$ satisfy the d-Hypercyclicity Criterion (Bès, Peris, 2007).

Linear Dynamics in $\mathcal{F}(M)$

Lipschitz d-Hypercyclicity Criterion

Corollary

If $N = 1$, Lipschitz Hypercyclicity Criterion (Abbar, Coine, Petitjean, 2021).

Just a glimpse

with Romuand Ernst and Quentin Menet (Mons, Belgium)

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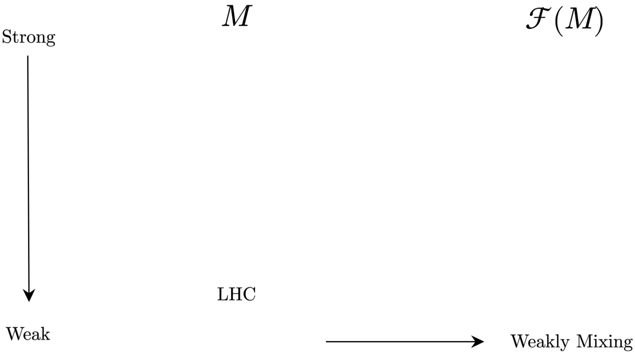

M

$\mathcal{F}(M)$

Strong

Weak

LHC

Weakly Mixing

Just a glimpse

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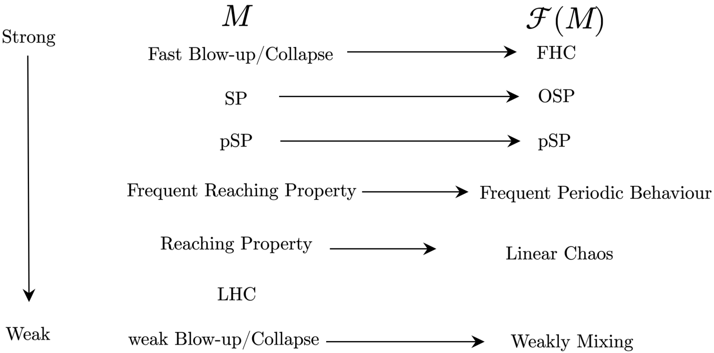

M → $\mathcal{F}(M)$

Strong

Fast Blow-up/Collapse → FHC

SP → OSP

pSP → pSP

Frequent Reaching Property → Frequent Periodic Behaviour

Reaching Property → Linear Chaos

LHC

weak Blow-up/Collapse → Weakly Mixing

Weak

Some references

- **J. Bès and A. Peris,** *Disjointness in hypercyclicity* **(2007).**
- **M. Murillo-Arcila and A. Peris,** *Chaotic Behaviour on Invariant Sets of Linear Operators* **(2015).**
- **A. Abbar, C. Coine, and C. Petitjean,** *On the Dynamics of Lipschitz Operators* **(2021).**
- **Ch. Cobollo and A. Peris,** *On disjoint dynamical properties and Lipschitz-free spaces* **(arXiv, 2024).**
- **K.-G. Grosse-Erdmann and A. Peris,** *Linear chaos* **(2011).**
- **N. Weaver,** *Lipschitz algebras 2nd ed* **(2018).**

The End

Thanks For Your Attention!