On Ramsey-type properties of the distance in nonseparable spheres - I

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Institute of Mathematics of the Polish Academy of Sciences

Minicourse Outline: dramatic structure

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  $\{C_i : i \in \mathcal{I}\}$ is the set of colors.
- Unordered pairs of X: $[X]^2 = \{\{x, y\} : x, y \in X \ \& \ x \neq y\}$.
- A coloring of pairs of X: $c : [X]^2 \to \{C_i : i \in \mathcal{I}\}$.
- A monochromatic set $M \subseteq X$ of a coloring c of pairs of X:

$$|c[[M]^2]| = 1.$$

- If $i \in I$, we say that $M \subseteq S$ is C_i-monochromatic if

$$c[[M]^2] = \{C_i\}.$$

- Examples: Suppose (X, d) is a metric space.
  1. $d' : [X]^2 \to \mathbb{R}_+$ given by $d'(\{x, y\}) = d((x, y))$ is a coloring.
  2. monochromatic sets of d' are equilateral subsets of X.
  3. $c : [X]^2 \to \{*, **\}$ given by $c(\{x, y\}) = *$ if $d(x, y) > 1$ and otherwise $c(\{x, y\}) = **$ is a coloring.
  4. $*$-monochromatic sets of c are $(1+)$-separated subsets of X.

The existence of infinite monochromatic sets. Ramsey theory.

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Theorem (Ramsey; 1930)

Suppose that X is an infinite set and $c : [X]^2 \to \{C_0, \ldots C_n\}$ for $n \in \mathbb{N}$. There is an infinite monochromatic $M \subseteq X$ (i.e., such that $c[[M]^2]$ is a singleton).

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Assume MA. *Suppose that $\mathbb{P}$ is a partial order of cardinality less than 2^ω. Either $\mathbb{P}$ admits an uncountable subset of pairwise incompatible elements or $\mathbb{P} = \bigcup_{n\in\mathbb{N}} \mathbb{P}_n$, where each $\mathbb{P}_n$ is centered.*

Definition

M - topological space. A coloring $c : [M]^2 \to \{0,1\}$ is open if $\{(x,y) \in M \times M : c(\{x,y\}) = 0\}$ is open in $M \times M$.
We say that OCA (Open Coloring Axiom) holds for M if for every open $c : [M]^2 \to \{0,1\}$ either

1. $M = \bigcup_{n\in\mathbb{N}} M_n$ where M_n is 1-monochromatic for each $n \in \mathbb{N}$ or
2. else there is an uncountable $N \subseteq M$ which is 0-monochromatic.

Theorem

- *(Todorcevic; 1989) It is consistent that* OCA *holds for every second countable regular space M. This is known just as* OCA.
- *(Feng; 1993)* OCA *holds for every analytic set.*

Theorem

Assume MA*. Suppose that $\mathbb{P}$ is a partial order of cardinality less than 2^ω. Either $\mathbb{P}$ admits an uncountable subset of pairwise incompatible elements or $\mathbb{P} = \bigcup_{n\in\mathbb{N}} \mathbb{P}_n$, where each $\mathbb{P}_n$ is centered.*

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