On Ramsey-type properties of the distance in nonseparable spheres - II

Piotr Koszmider

Institute of Mathematics of the Polish Academy of Sciences

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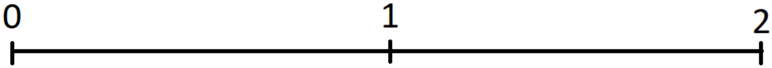

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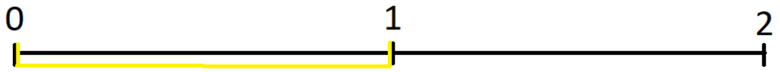

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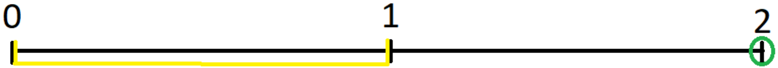

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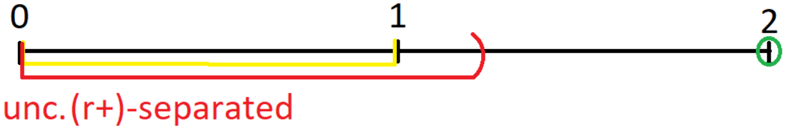

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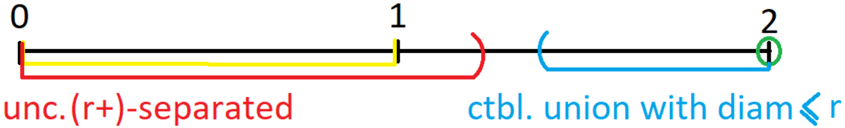

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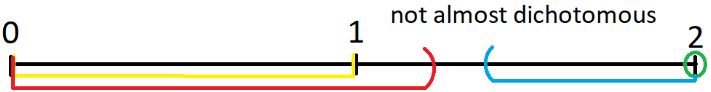

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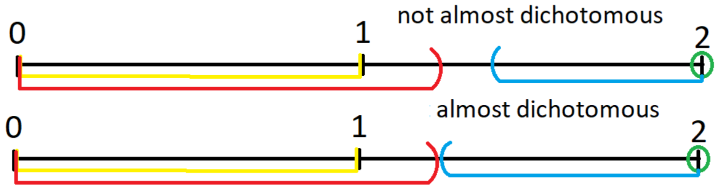

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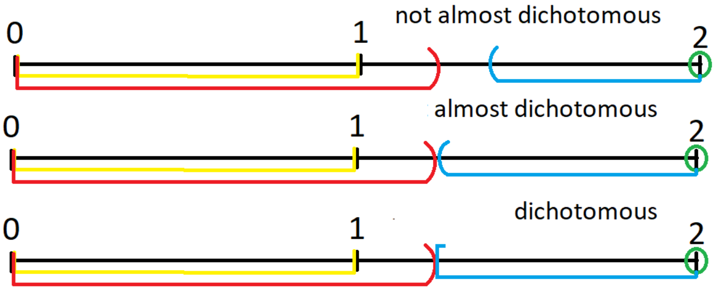

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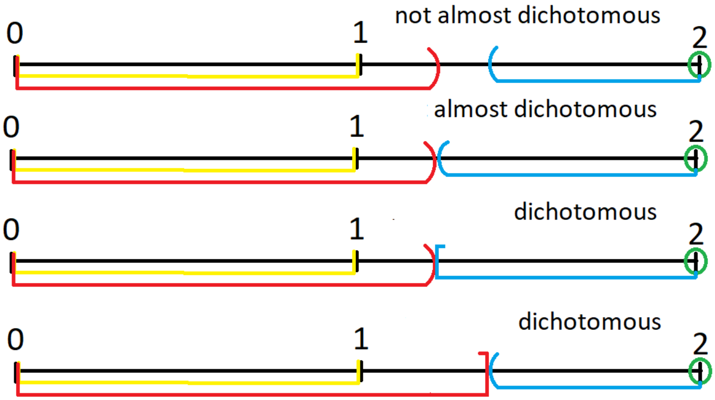

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- $S_n \cap \mathcal{Y}$ is uncountable for some $n \in \mathbb{N}$. Then $S_n \cap \mathcal{Y}$ is ε-approximately r-equilateral.
- By scaling and translating we get an uncountabe ε-approximately 1-equilateral set.

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Properties of Banach spaces with (almost) dichotomous spheres

Theorem

If a Banach space has dichotomous unit sphere and admits an uncountable Auerbach system, then it admits an uncountable $(1+)$-separated set.

Theorem

If a Banach space has an almost dichotomous unit sphere, then it admits uncountable ε-approximately 1-equilateral sets for every $\varepsilon > 0$.

Proof.

- Let $r = \sup(K^+(S_\mathcal{X})) = \inf(\Sigma(S_\mathcal{X}))$. By the Riesz lemma $r \geq 1$.
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Under MA *and the negation of* CH *the unit spheres of all nonseparable Banach spaces of densities less than* 2^ω *of the form* $C_0(K)$ *for* K *locally compact are dichotomous.*

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Question (B)

Is there in ZFC *a nonseparable hyperlateral Banach space?*

Definition

Suppose that M is a set and $d : [M]^2 \to \mathbb{R}_+$. We say that M is hyperlateral if for every uncountable separated $N \subseteq M$ there is $\varepsilon > 0$ such that N does not contain an uncountable ε-approximately equilateral subset.

Proposition

Suppose that $\mathcal{X}$ is a nonseparable Banach space. Then the following are equivalent:

1. *$\mathcal{X}$ is hyperlateral.*
2. *$S_{\mathcal{X}}$ is hyperlateral.*
3. *for some $\varepsilon > 0$ there is no uncountable $\mathcal{Y} \subseteq S_{\mathcal{X}}$ such that $1 - \varepsilon < \|y - y'\| < 1 + \varepsilon$ for every distinct $y, y' \in \mathcal{Y}$.*

Question (B)

Is there in ZFC a nonseparable hyperlateral Banach space?

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