Universidad de Murcia | Departamento Matemáticas

Variational Analysis and James's Theorem

José Orihuela

New Perspectives in Banach spaces and Banach lattices.
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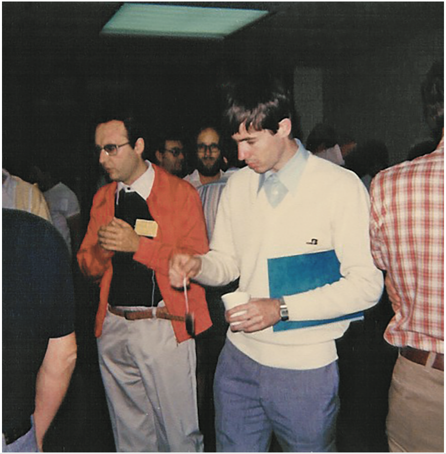
Coauthors and main publications

- J. Orihuela and M. Ruiz Galán *A coercive and nonlinear James's weak compactness theorem* Nonlinear Analysis 75 (2012) 598-611.
- B. Cascales, J. Orihuela and A. Pérez: *One-sided James Compactness Theorem*, J. Math. Anal. Appli. 445, Issue 2, 1267-1283 (2017).
- J. Orihuela: *Conic James' Compactness Theorem*, Journal of Convex Analysis (2018)(3), 1335–1344.
- F. Delbaen and J. Orihuela *Mackey's constraints for James's Compactness Theorem and Risk Measures*, Journal Math. Anal. Appli, 485-1 (2020), https://doi.org/10.1016/j.jmaa.2019.123764
- F. Delbaen and J. Orihuela *On the range of the subdifferential in non reflexive Banach spaces*, Journal Functional Anal., 281-2, (2021) https://doi.org/10.1016/j.jfa.2020.108915
- F. Delbaen and J. Orihuela *A multiset version of James's Theorem*, Journal Functional Analysis, (2022).
- F. Delbaen and J. Orihuela *Nonlinear James's* w^**-compactness theorem.* Work in progress, 2024.

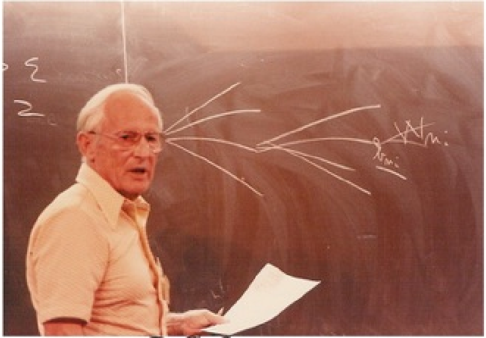
Prof. Dr. Freddy Delbaen

Jean Bourgain and Freddy Delbaen, Kent 1979



Charles Stegall: Applications of a Descriptive Topology in Functional Analysis

R.C. James



R.C. James

Delbaen and Schachermayer questions

Question (W. Schachermayer)

Let us fix a proper function

$$\alpha : \mathbb{L}^1(\Omega, \mathcal{F}, \mathbb{P}) \to (-\infty, +\infty]$$

When the minimization problem

$$\min\{\alpha(X) + \mathbb{E}[Y \cdot X] : X \in \mathbb{L}^1(\Omega, \mathcal{F}, \mathbb{P})\}$$

has solution for all $Y \in \mathbb{L}^\infty(\Omega, \mathcal{F}, \mathbb{P})$?

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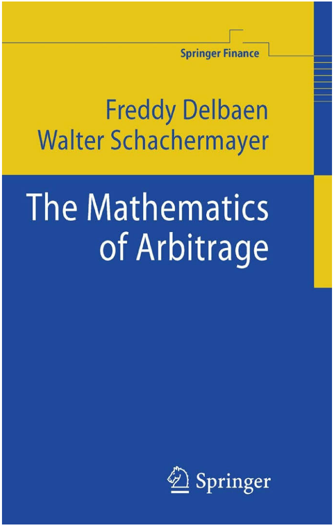
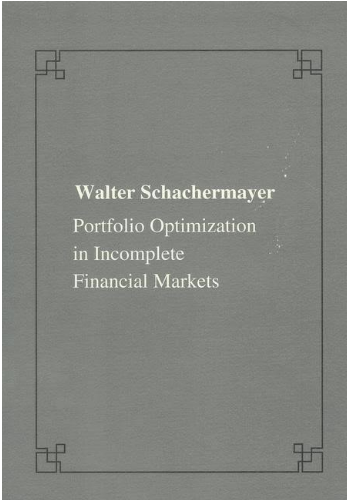
Question (F. Delbaen)

Let C be a convex, bounded and closed, but not weakly compact subset of the Banach space $\mathbb{L}^1(\Omega, \mathcal{F}, \mathbb{P})$ with $0 \notin C$.
Is it possible to find a linear functional $Y \in \mathbb{L}^\infty(\Omega, \mathcal{F}, \mathbb{P})$ not attaining its minimum on C but that stays strictly positive on C?

Prof. Dr. Walter Schachermayer


Springer Finance
Freddy Delbaen
Walter Schachermayer
The Mathematics of Arbitrage
Springer


Walter Schachermayer
Portfolio Optimization in Incomplete Financial Markets

The answers

Theorem (F. Delbaen and J. Orihuela)

Let A be a convex, closed, bounded but non weakly compact subset of a Banach space E such that $0 \notin A$. Let us fix a non-void open set Ω in the Makey dual $(E^*, \tau(E^*, E))$.
Then there is a continuous linear form $x_0^* \in \Omega$ which doest not attains supremum on A and such that

$$\sup x_0^*(A) < 0$$

Maximizing $\{x^*(x) - \alpha(x) : x \in E\}$

Theorem (F. Delbaen and J. Orihuela)

Let E be a Banach space,

$$\alpha : E \to (-\infty, +\infty]$$

be a proper and bounded below function such that $\partial\alpha(E)$ has non empty interior in E^* for the Mackey topology $\tau(E^*, E)$, then the level sets

$$\{\alpha \leq c\}$$

are relatively weakly compact for all $c \in \mathbb{R}$. If in addition the function α has a domain with non-empty norm interior, the Banach space must be reflexive.

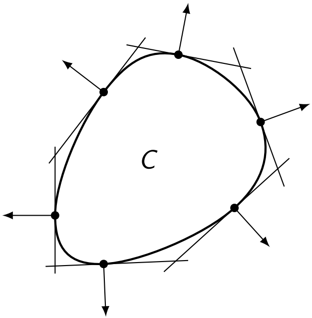
Let X be a (real) Banach space.

Theorem (James, 1964)

Let $C \subset X$ be a bounded, convex and closed set such that

$$\forall\, x^* \in X^* \quad \exists\, c \in C \text{ with } \langle x^*, c\rangle = \sup(x^*, C)$$

Then, C is w-compact.



A new measure of non-weak compactness

Theorem

Let A be a bounded subset of a Banach space E. Then A is weakly relatively compact if, and only if, for every bounded sequence $\{x_n^*\}_{n\geq 1}$ in E^* we have

$$\mathrm{dist}_{\|\cdot\|_A}(L\{x_n^*\}, \mathrm{co}\{x_n^* : n \geq 1\}) = 0.$$

where we are denoting with $L\{x_n^*\}$ the set of all w^*-cluster points of the bounded sequence $\{x_n^*\}$ in E^*,
and

$$\|x^*\|_A := \sup\{|x^*(a)| : a \in A\}$$

for every $x^* \in E^*$.

Connection with Pryce's arguments for the general case

Theorem

Let E be a Banach space, A a bounded subset of E with $A = -A$, $\{x_n^*\}_{n\geq 1}$ a bounded sequence in the dual space E^*, and D its norm-closed linear span in E^*. Then there exists a subsequence $\{x_{n_k}^*\}_{k\geq 1}$ of $\{x_n^*\}_{n\geq 1}$ such that

$$S_A\left(x^* - \liminf_k x_{n_k}^*\right) = S_A\left(x^* - \limsup_k x_{n_k}^*\right) =$$

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$$= \operatorname{dist}_{\|\cdot\|_A}(x^*, L\{x_{n_k}^*\})$$

for all $x^* \in D$.

Theorem (James-Pryce undetermined function technique)

Let X be a nonempty set, $\{h_j\}_{j\geq 1}$ a bounded sequence in $\ell^\infty(X)$, and $\delta > 0$ such that

$$S_X\left(h - \limsup_j h_j\right) = S_X\left(h - \liminf_j h_j\right) \geq \delta,$$

whenever $h \in \mathrm{co}_\sigma\{h_j : j \geq 1\}$. Then there exists a sequence $\{g_i\}_{i\geq 1}$ in $\ell^\infty(X)$ with

$$g_i \in co_\sigma\{h_j : j \geq i\}, \quad \text{for all } i \geq 1,$$

and there exists $g_0 \in \mathrm{co}_\sigma\{g_i : i \geq 1\}$ such that for all $g \in \ell^\infty(X)$ with

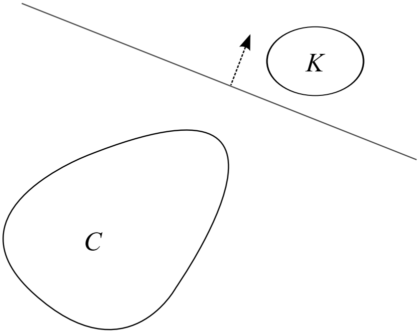
$$\liminf_i g_i \leq g \leq \limsup_i g_i \quad \text{on } X,$$

the function $g_0 - g$ doest not attain its supremum on X .

One-side James' Theorem

$C \subset E$ convex closed bounded
$K \subset E$ convex weakly compact
$C \cap K = \emptyset$

> $x^* \in E^*$ with
>
> $$\sup(x^*, C) < \inf(x^*, K)$$



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Hypothesis 1:
Every $x^* \in E^*$ with

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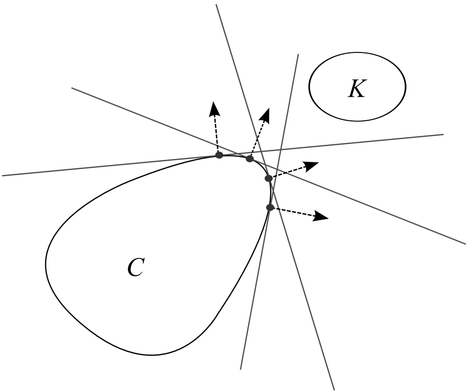

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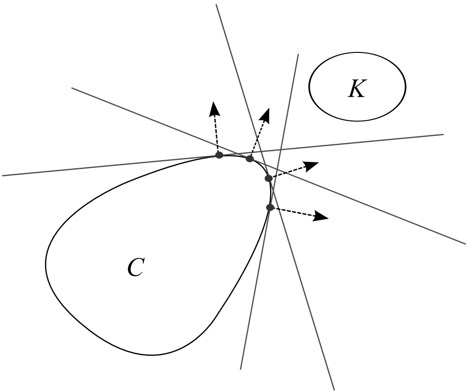

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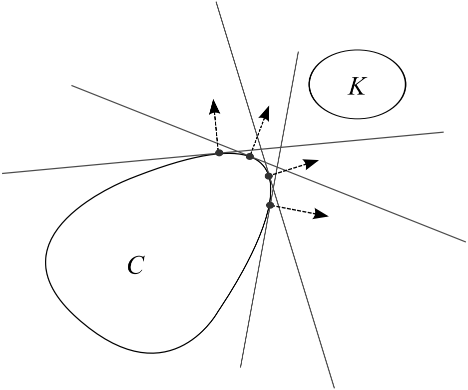
Thesis: C is weakly compact.



One-sided plus Mackey's constraints Case

Theorem (Joint work with Freddy Delbaen)

Let A be a convex, closed and bounded subset of a Banach space E which is assumed non to be weakly compact with $0 \notin A$. Let us fix a relatively weakly compact subset D in $(E, \sigma(E, E^*))$ together with an absolutely convex and weakly compact subset W in $(E, \sigma(E, E^*))$ and a functional $z_0^* \in E^*$ with

$$\inf z_0^*(A) > 0, \inf z_0^*(D) > 0 \textit{ and } \epsilon > 0.$$

Then there is a linear form $z^* \in E^*$ such that

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Then there is a linear form $z^* \in E^*$ such that

1. $a \to \langle z_0^* + z^*, a \rangle$ *does not attain its infimum on A,*
2. $\sup_{w \in W} |z^*(w)| < \epsilon$, *and*
3. $\inf(z_0^* + z^*)(A) > 0, \inf(z_0^* + z^*)(D) > 0$

Theorem (Delbaen - Orihuela)

Let $u_1 : \mathbb{L}^\infty(\Omega, \mathcal{F}, \mathbb{P}) \to \mathbb{R}$ be a Fatou coherent monetary utility function. Suppose that u_1 is not the essential infimum function. The following are equivalent:

1. *u_1 is a Lebesgue monetary utility function*
2. *$u_1 \Box u_2$ is Fatou for all Fatou coherent utility functions u_2*
3. *$u_1 \Box u_2$ is Lebesgue for all Fatou coherent utility functionu_2*

The Unbounded Case

Theorem (Delbaen - Orihuela)

Let C be a closed, convex unbounded subset in the Banach space E and D be weakly compact subset of E such that every bounded set $Z \in E^*$ satisfies that

$$\sup\{z^*(c) : c \in C, z^* \in Z\} < +\infty$$

whenever $\sup\{z^*(d) : d \in D, z^* \in D\} < 0.$

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1. $\sup_{w \in W} |z^*(w)| < \epsilon$, *and*
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1. $\sup_{w \in W} |z^*(w)| < \epsilon$, *and*
2. $\sup\{(z_0^* + z^*)(d) : d \in D\} < 0$, *and so*
3. $\sup\{(z_0^* + z^*)(c) : c \in C\} < +\infty$ *but this supremum is not attained.*

Maximizing $\{x^*(x) - \alpha(x) : x \in E\}$

Theorem (Delbaen - Orihuela)

Let E be a Banach space, $\alpha : E \to (-\infty, +\infty]$ be a proper and bounded below function.

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Theorem (Delbaen - Orihuela)

Let E be a Banach space, $\alpha : E \to (-\infty, +\infty]$ be a proper and bounded below function.
If $\partial\alpha(E)$ has non empty interior in E^* for the Mackey topology $\tau(E^*, E)$, then the level sets $\{\alpha \leq c\}$ are relatively weakly compact for all $c \in \mathbb{R}$.

Corollary

Let E be a real Banach space and let $\alpha : E \longrightarrow \mathbb{R} \cup \{+\infty\}$ be a bounded below function such that $\mathrm{dom}(\alpha)$ has nonempty norm–interior and for all $x^* \in U$ there exists $x_0 \in E$ with

$$\alpha(x_0) - x^*(x_0) = \inf_{x \in E} (\alpha(x) - x^*(x)), \tag{1}$$

where U is a non void $\tau(E^*, E)$-open set,

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where U is a non void $\tau(E^*, E)$-open set, then E is a reflexive Banach space.

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where U is a non void $\tau(E^*, E)$-open set, then E is a reflexive Banach space. Moreover, the minimization problem (1) has a solution for all $x^* \in E^*$.

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where U is a non void $\tau(E^*, E)$-open set, then E is a reflexive Banach space. Moreover, the minimization problem (1) has a solution for all $x^* \in E^*$.
In particular, if we have a monotone and symmetric map $\Phi : E \longrightarrow E^*$ such that $\Phi(E)$ has non empty interior for the Mackey topology $\tau(E^*, E)$, the Banach space E must be reflexive and $\Phi(E) = E^*$

Last news on James' theorem

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Let A be closed, convex and bounded subset of a Banach space E. If A is not weakly relatively compact, is there $x^* \in E^*$ such that $x^*(A) = (\alpha, \beta)$?

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Theorem (A Dichotomous James' Theorem, Delbaen - Orihuela)

Let $A \subset E$ be a bounded subset of a weakly sequentially complete Banach space E. If every $x^* \in E^*$ either attains its supremum or infimum on A, then A is weakly relatively compact.

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The hypothesis of weakly sequentially completenes can not be removed

Theorem (James - Pryce - Delbaen - Orihuela)

Let $\{h_j\}_{j\geq 1}$ *be a uniformly bounded sequence in* $\mathbb{R}^{X\cup Y}$. *Let*

$$0 < A < 1 \leq K$$

be positive real numbers such that for all $h_0 \in co_\sigma\{h_j : j \geq 1\}$*:*

$$0 < A \leq S_X(h_0 - \limsup_j h_j) = S_X(h_0 - \liminf_j h_j) \leq K < \infty$$

and

$$0 < A \leq S_Y(h_0 - \limsup_j h_j) = S_Y(h_0 - \liminf_j h_j) \leq K < \infty.$$

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Then there is a pseudo-subsequence $\{g_i\}_{i\geq 1}$ *of* $\{h_j\}_{j\geq 1}$, *and* $g_0 \in co_\sigma\{g_i : i \geq 1\}$, *such that for every* $\hat{g}$ *satisfying for every* $x \in X$

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$$\liminf g_i(x) \leq \hat{g}(x) \leq \limsup g_i(x),$$

we have that $g_0 - \hat{g}$ *does not attain its supremum neither on* X *nor on* Y*.*

Multiset James's Theorem

Corollary (Delbaen - Orihuela)

Let A and B be closed, bounded and convex subsets of Banach space E. If there are vectors $x_0^{**} \in \overline{A}^{\sigma(E^{**},E^{*})} \setminus E, y_0^{**} \in \overline{B}^{\sigma(E^{**},E^{*})} E^{**} \setminus E$ with

$$[x_0^{**}, y_0^{**}] \cap E = \emptyset,$$

then there exist $x^* \in E^*$ such that x^* does not attain its supremum neither on A or on B.

Multiset James's Theorem

Theorem (Delbaen - Orihuela)

Let $A_1, A_2, \cdots A_p$ be closed, bounded and convex subsets of Banach space E. If there are vectors $x_1^{**}, x_2^{**}, \cdots x_p^{**} \in E^{**} \setminus E$ with

$$x_i^{**} \in \overline{A_i}^{\sigma(E^{**},E^*)} \setminus A_i : i = 1, 2, \cdots, p$$

and

$$\mathrm{co}(\{x_i^{**} : i = 1, 2, \cdots, p\}) \cap E = \emptyset,$$

then there exists $x^* \in E^*$ such that x^* does not attain its supremum on any A_i for $i = 1, 2, \cdots, p$.

Multiset James's Theorem

Theorem (Delbaen - Orihuela)

Let $\{A_1, A_2, \cdots A_p\}$ be a finite family of closed, bounded, convex but not weakly compact subsets of a weakly sequentially complete Banach space E. Then there exist $x^* \in E^*$ such that x^* does not attain its supremum on any A_i for $i = 1, 2, \cdots, p$.

Nonlinear w*-James's compactness Theorem

Theorem

Let E be a Banach space without copies of ℓ^1 together with a w-K analytic subset $A \subset E^*$. Let $B \subset E^*$ be such that $A \subset B \subset \overline{A}^{\|\cdot\|}$ and $D \subset E^*$ convex and weakly compact set with*

$$(-D) \cap \overline{\mathrm{co}(B \cup \{0\})}^{\|\cdot\|} = \emptyset. \tag{2}$$

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Let us assume that for $x \in E$ with $x(d^*) < 0$ for every $d^* \in D$, we have

$$\sup\{x(c^*) : c^* \in B\} = x(b^*) \tag{3}$$

for some $b^* \in B$.

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Let us assume that for $x \in E$ with $x(d^*) < 0$ for every $d^* \in D$, we have

$$\sup\{x(c^*) : c^* \in B\} = x(b^*) \qquad (3)$$

for some $b^* \in B$. Then we have that

$$\overline{\mathrm{co}(B)}^{\omega^*} \subset \overline{\mathrm{co}(B)}^{\|\cdot\|} + \Lambda_D.$$

Nonlinear w*-James's compactness Theorem

Corollary

Let E be a Banach space without copies of ℓ^1 and $B \subset E^*$ be a norm closed convex and w*-K *analytic set, with $0 \in B$ and such that $B + B \subset B$. Let us assume there is a weakly compact convex set $D \subset B$ with $(-D) \cap B = \emptyset$ such that for $x \in E$ with $x(d^*) < 0$ for every $d^* \in D$, we have*

$$\sup\{x(c^*) : c^* \in B\} = 0 \tag{4}$$

Then B is going to be w*-*closed, i.e.:*

$$\overline{B}^{\omega^*} \subset B.$$

Nonlinear w*-James's compactness Theorem

Theorem

Let E be a Banach space without copies of $\ell^1(\mathbb{N})$ and let

$$\alpha : E^* \longrightarrow \mathbb{R} \cup \{+\infty\}$$

be a convex proper and norm lower semicontinuous map with a w-K-analytic subset $A \subset \mathrm{dom}\,(\alpha)$ with $\mathrm{dom}\,(\alpha) \subset \overline{A}^{\|\cdot\|}$, and such that*

$$\text{for all } x \in E, \quad x - \alpha \text{ attains its supremum on } E^*.$$

Then α is w-lower semicontinuous and for every $\mu \in \mathbb{R}$, the sublevel set $\alpha^{-1}((-\infty, \mu])$ is w*-compact.*

Nonlinear w*-James's compactness Theorem

Theorem

Let E be a Banach space and $B \subset E^*$, let A and D be weakly countably determined subsets of E^* with $B \subset \overline{A}^{\|\cdot\|}$, and D bounded, w-closed and convex with $0 \notin D$. If for every $x \in E$, with $x(d^*) < 0$ for every $d^* \in D$, we have that*

$$\sup\{x(c^*) : c^* \in B\} = x(b^*) \tag{5}$$

for some $b^* \in B$, then

$$\overline{\mathrm{co}(B)}^{\mathrm{w}^*} \subset \overline{\mathrm{co}(B) + \Lambda_D}^{\|\cdot\|}$$

Corollary

Let E be a Banach space and $B \subset E^*$ be a norm closed convex and weakly countably determined subset, with $0 \in B$ and such that $B + B \subset B$. Let us assume there is a bounded and w**-closed set $D \subset B$ with $0 \notin D$ and such that for $x \in E$ with $x(d^*) < 0$ for every $d^* \in D$, we have*

$$\sup\{x(c^*) : c^* \in B\} = 0 \tag{6}$$

Then we have that B is going to be w**-closed, i.e.:*

$$\overline{B}^{\omega^*} \subset B.$$

Nonlinear w*-James's compactness Theorem

Theorem

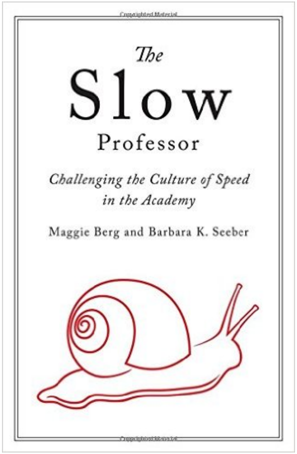
Let E be a Banach space and let

$$\alpha : E^* \longrightarrow \mathbb{R} \cup \{+\infty\}$$

be a convex, proper and norm lower semicontinuous map with a weakly countably determined subset $A \subset \operatorname{dom}(\alpha)$ such that $\operatorname{dom}(\alpha) \subset \overline{A}^{\|\cdot\|}$. Let us assume that

$$\text{for all } x \in E, \quad x - \alpha \text{ attains its supremum on } E^*,$$

then α is w*-*lower semicontinuous and for every $\mu \in \mathbb{R}$, the sublevel set $\alpha^{-1}((-\infty, \mu])$ is* w*-*compact.*



Slow Science Manifesto

THANKS A LOT FOR YOUR ATTENTION ... !!

Nonlinear w*-James's compactness Theorem

Theorem (γ-Conic Godefroy's Theorem)

Let E be a Banach space without copies of l^1. Let D be a convex and w^*-closed subset with $0 \notin D$ and $B \subset E^*$ a nonempty set satisfying that for each $x \in E$ such that $x(D) < 0$ there is $b^* \in B$ with $\langle x, b^* \rangle = \sup(x, B)$. Then, we have that

$$\overline{\mathrm{co}(B)}^{\omega^*} \subset \overline{\mathrm{co}(B) + \Lambda_D}^{\gamma(E^*,E)}.$$

Nonlinear w*-James's compactness Theorem

Theorem

Let E be a Banach space without copies of $\ell^1(\mathbb{N})$. If $\alpha : E^* \longrightarrow \mathbb{R} \cup \{+\infty\}$ is a convex, proper and $\gamma(E^*, E)$-lower semicontinuous map such that

$$\text{for all } x \in E, \quad x - \alpha \text{ attains its supremum on } E^*, \tag{7}$$

then α is w-lower semicontinuous and for every $\mu \in \mathbb{R}$, the sublevel set $\alpha^{-1}((-\infty, \mu])$ is w*-compact.*
If E is a Banach lattice without copies of $\ell^1(\mathbb{N})$and we assume that $\alpha(x^*) \leq 0$ for $x^* \in E^*_-$, then condition (7) can be relaxed to ask for

$$\text{for all } x \in E_+, \quad x - \alpha \text{ attains its supremum on } E^*, \tag{8}$$

and we also get the w-lower semicontinuouty for α and the fact that its level sets are w*-compact.*

Corollary

Let E be a Banach space and $B \subset E^*$ be a norm closed convex and weakly countably determined subset, with $0 \in B$ and such that $B + B \subset B$. Let us assume there is a weakly countably determined, convex, bounded and w**-closed set $D \subset B$ with $0 \notin D$ and such that for $x \in E$ with $x(d^*) < 0$ for every $d^* \in D$, we have*

$$\sup\{x(c^*) : c^* \in B\} = 0 \tag{9}$$

Then we have that B is going to be w**-closed, i.e.:*

$$\overline{B}^{\omega^*} \subset B.$$

Theorem

Let B, D be subsets of a dual Banach space E^* such that D is assumed to be a $\sigma(E^*, E)$-closed convex subset with $0 \notin D$. Given

$$x \in E \text{ such that : } x(d^*) < 0 \text{ for every } d^* \in D,$$

there is $b^* \in B$ with

$$\langle x, b^* \rangle = \sup \langle x, B \rangle,$$

and $B \subset \cup_{n=1}^{\infty} K_n$ for some family of w^*-compact convex subsets of E^*, then we have:

$$\overline{\mathrm{co}(B)}^{\omega^*} \subset \overline{\mathrm{co}(\cup_{n=1}^{\infty} K_n) + \Lambda_D}^{\|\cdot\|}.$$

Corollary

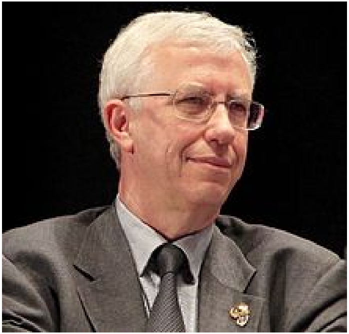
Let E be a Banach space without copies of ℓ^1 and $B \subset E^*$ be a norm closed convex and w^*-K analytic set, with $0 \in B$ and such that $B + B \subset B$. Let us assume there is a weakly compact convex set $D \subset B$ with $(-D) \cap B = \emptyset$ such that for $x \in E$ with $x(d^*) < 0$ for every $d^* \in D$, we have

$$\sup\{x(c^*) : c^* \in B\} = 0 \tag{10}$$

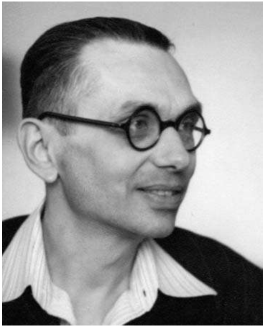
Then B is going to be w^*-closed, i.e.:

$$\overline{B}^{\omega^*} \subset B.$$

Lehman Brothers default



Mathematics and uncertainty



Kurt Gödel

Prof. Dr. Walter Schachermayer

Walter Schachermayer

Fakultät für Mathematik,
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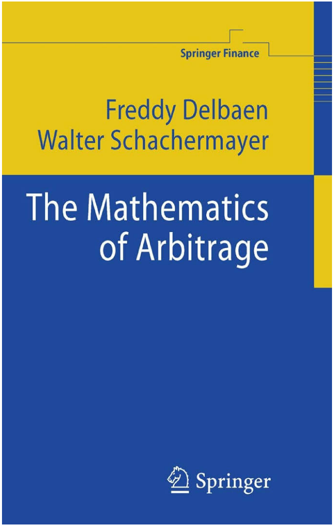
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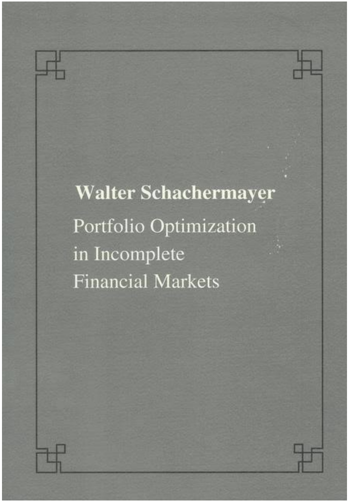
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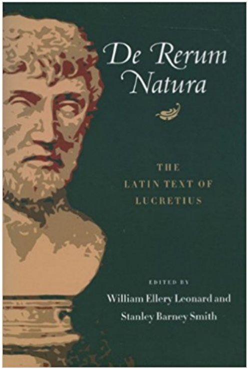
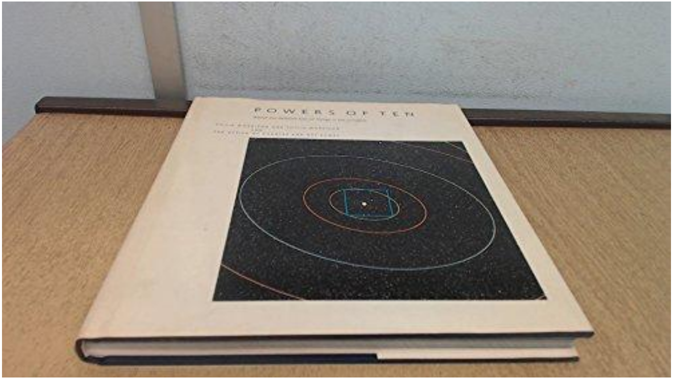
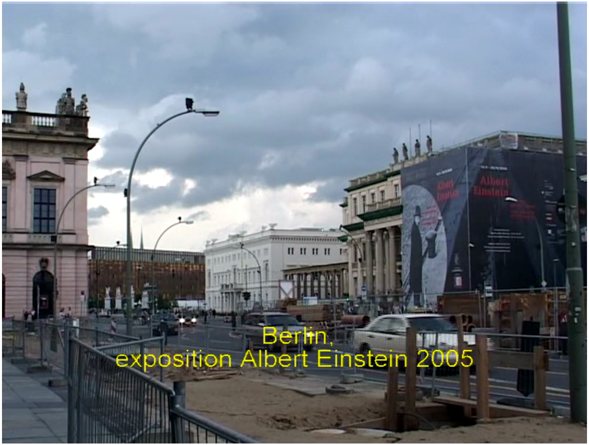


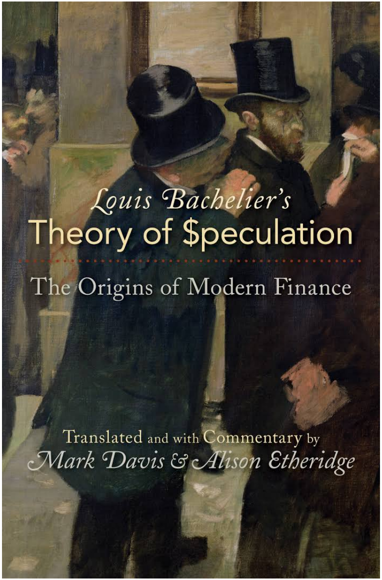
Titus Lucretius Carus (99-55 antes de Cristo)



Powers of ten.

Brownian Motion movie

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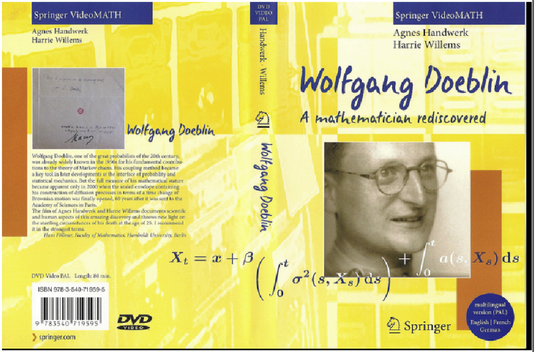
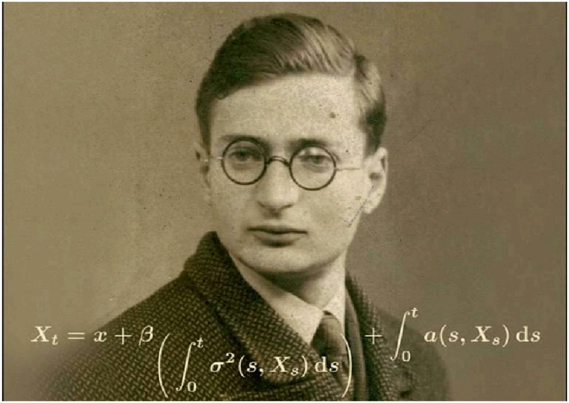
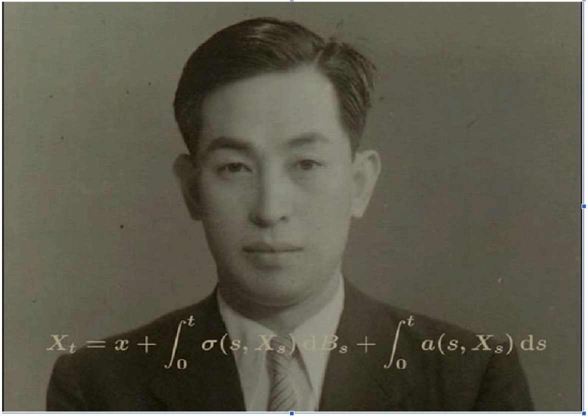


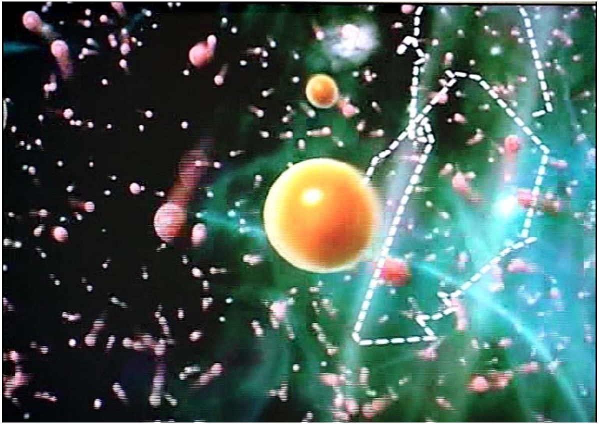
Doeblin-Itô (1935-2000)



Wolfgang Doeblin y su fórmula



Kiyoshi Itô y su fórmula



Path of a Brownian Motion

R. Merton, M. Sholes y F. Black

Paul Embrechets, ETH Zurich. *Extremal events researcher who advised on the risks of the copula formula what killed Wall Street in 2007*

$$\Pr[T_A<1, T_B<1] = \phi_2(\phi^{-1}(F_A(1)), \phi^{-1}(F_B(1)), \gamma)$$

Here's what killed your 401(k) *David X. Li's Gaussian copula function as first published in 2000. Investors exploited it as a quick—and fatally flawed—way to assess risk. A shorter version*

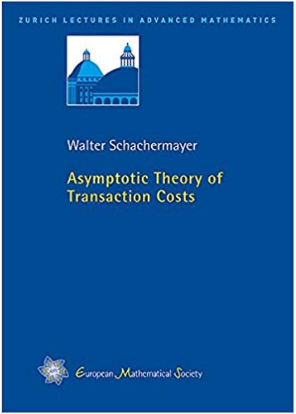
David X. Li
Illustration: David A. Johnson

En el mundo financiero muchos "quants" ven solo números ante ellos y olvidan sobre la realidad concreta que dichos números representan. Piensan que pueden modelar con unos pocos años de datos y calculan probabilidades para sucesos que pueden ocurrir solamente una vez en 10.000 años. Entonces los inversores invierten sobre la base de dichas probabilidades, sin pararse a preguntarse si los números tienen algún sentido.

D. Li "The most dangerous part is when people believe everything coming out of it" (sobre su modelo....)

David X. Li

Felix Salmon: The formula that killed Wall Street


ZURICH LECTURES IN ADVANCED MATHEMATICS
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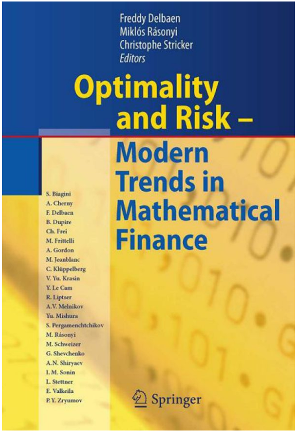
Asymptotic Theory of Transaction Costs
European Mathematical Society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